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paper as being a long run situation. But his interpretation is again incorrect. As is clearly stated, my paper is concerned with the short-run behavior of real wages and output within a framework of short-run theories of employment and output, so that points off the labor demand schedule mean those points off the short-run schedule. Therefore, the use of the regression results obtained by the "short-run employment function model" discussed in his note—which is a short-run equilibrium model—is not appropriate for countries

experiencing short-run disequilibrium conditions of the type that are discussed in my paper.

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MONEY AND THE PRODUCTION FUNCTION—A REPLY TO BOYES AND KAVANAUGH

Allen Sinai and Houston H. Stokes*

I. Introduction

In a recent note, Boyes and Kavanaugh (BK, 1979) make two principal arguments: first, that the "ideal" two factor production function (one free from specification error) is constant elasticity of substitution (CES) and second, that including real cash balances in the production function is a misspecification.

We demonstrate that (1) contrary to their claim, the BK version of the CES production function is not free from specification error; (2) the particular form of the CES and the two stage linear estimation method used are responsible for the BK conclusion that the CES is the appropriate functional form; (3) real money balances are a significant input even in the CES form suggested, and when such a specification is corrected for serial correlation with a generalized least squares (GLS) procedure it converges to a Cobb-Douglas (CD) function; and (4) the likely specification error in the CD function with money balances is simultaneity, which does not necessarily invalidate the role of money balances in production. Finally, when a new set of data, disaggregated to the nonfinancial corporate sector, is confronted with the hypothesis that real money balances belong in the production function, further support is obtained.

II. Specification Error in the Aggregate Production Function

BK employ a series of tests for specification error due to Ramsey (1969, 1974) and applied in Ramsey and Zarembka (1971). One of them is BAMSET, as de-

scribed in Ramsey (1969, pp. 34–37) and BK (1979), designed to detect heteroscedasticity. BK indicate "the BAMSET test rejects the CD form in all cases but one" and for the CES including real cash balances "BAMSET indicates rejection only for the model with M3." BK also state "none of the specification error tests indicates rejection of the two-factor CES model."

A problem with the BAMSET procedure, however, is that the most appropriate best linear unbiased scalar (BLUS) base is not selected and thus the test loses power. In an alternative approach not subject to this difficulty, Theil (1971, ch. 5), the data are first resorted against each explanatory variable as well as the original order and the BLUS residual base is selected as the middle K observations of the resorted data matrix. Using this procedure, a different set of BLUS residuals is obtained for every explanatory variable and then used to form a specific test for heteroscedasticity relevant to that variable. We report these more powerful tests, the statistics of which have an F -distribution, in table 1.¹

The two-factor CES, which BK offer as the "ideal" form (reported as CES I in table 1), is found to have significant levels of heteroscedasticity with respect to the original order (at the 0.9945 level) and with respect to capital and labor (at levels of significance 0.99962 and 0.9982, respectively). In addition, the serial correlation of the residuals indicated for the alleged ideal form, which BK did not report, is very high ($D.W. = 0.8788$). We tested the three-factor CES containing,

¹ In the presence of serial correlation, neither the power of the Theil test nor the BAMSET test is completely known. The advantage of the Theil test is its specificity on each explanatory variable and the resorting to obtain a more appropriate BLUS set of residuals. Correction for serial correlation in the application of the Theil procedure would be of little help because the data are resorted.

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TABLE 1.—RESIDUAL HETEROSCEDASTICITY TESTS APPLIED TO THE BOYES-KAVANAUGH
CONDITIONALLY ESTIMATED (OLS) CES PRODUCTION FUNCTION, 1927–1967

$$Y_i^{pr} = \delta_1 K_i^\rho + \delta_2 L_i^\rho + \delta_3 RM_i^\rho + u_{2i}, i = 1, \dots, n$$

| Model | Data Sorted Against | | | | D.W. |
|--|------------------------------------|--------------------------------------|--------------------------------------|------------------------------------|-------|
| | Time | lnK | lnL | ln(M/P) | |
| CES 1 $\rho = .01, v = 1.85$ no M/P term | $F(13,13) = 4.474$ Prob = .9945 | $F(18,19) = 5.248$ Prob = .99962 | $F(18,19) = 4.110$ Prob = .9982 | | .8788 |
| CES 2 $\rho = .05, v = 1.8$ contains M1/P | $F(13,13) = 5.665$ Prob = .9982 | $1/F(18,18) = 6.023$ Prob = .9998 | $1/F(18,18) = 7.476$ Prob = .9999 | $F(18,18) = 1.579$ Prob = .8294 | .7634 |
| CES 3 $\rho = -.05, v = 1.9$ contains M2/P | $F(13,13) = 4.666$ Prob = .9954 | $1/F(18,18) = 4.887$ Prob = .9992 | $1/F(18,18) = 5.750$ Prob = .9997 | $F(18,18) = 1.527$ Prob = .8112 | .7648 |
| CES 4 $\rho = -.05, v = 1.9$ contains M3/P | $F(13,13) = 4.307$ Prob = .9935 | $1/F(18,18) = 5.173$ Prob = .9995 | $1/F(18,18) = 5.962$ Prob = .9998 | $F(18,18) = 1.574$ Prob = .8278 | .7684 |

Note: For a complete description of the data, see Sinai and Stokes (1972, p. 296). The equations are those reported by Boyes-Kavanaugh (1979) as (II) and in their table 1. The Theil (1971, ch. 5) test for heteroscedasticity involves first sorting the data with respect to each variable (and the original order), then calculating BLUS residuals where the base is the middle K observations of the resorted data set. $F(n1.n2)$ is formed as the sum of the first half of the $N - K$ BLUS residuals squared divided by the sum of the last half of the $N - K$ BLUS residuals squared. Prob indicates the significance level for the F -statistic. D.W. = Durbin-Watson statistic.

respectively, M1, M2 and M3 (CES 2, CES 3, CES 4 in table 1) and continued to find highly significant levels of heteroscedasticity with respect to the original order and for capital and labor. But we did not find significant heteroscedasticity with respect to real money balances. Thus, contrary to the BK assertion, the two-factor CES production function was not ideal.

III. CES or CD?

The estimation method used by BK, as described in Zarembka (1974), involved a two-stage procedure where a grid of values for v (the degree of returns to scale) and ρ (the substitution parameter) was searched to permit linear estimation. The efficiency parameter was constrained to unity, thus eliminating the constant.² In view of the fact that the parameters of the CES have been found to be "highly sensitive to . . . methods of estimation,"³ we performed a nonlinear estimation of the BK two-factor CES with a subroutine (GAUSHAUS), based on the Marquardt algorithm.⁴ The results are reported as CES 5 in table 2.

High levels of serial correlation (D.W. = 0.8376) are still indicated and very low t -statistics appear for all but the returns to scale parameter v , offering scant support for the BK "ideal" specification. Diagnostic checking indicates a high 0.9963 correlation coefficient between the two distribution parameters (δ_1, δ_2) and between δ_1, δ_2 and the substitution parameter ρ of

–0.9978 and –0.9998, respectively. These findings suggest that severe collinearity is responsible for the large standard errors of the regression coefficients in CES 5.⁵ A more appropriate functional specification might be to constrain the sum of the distribution parameters at unity.

This change is made in CES 6 of table 2. In addition, the assumed value of unity for the scaling parameter α is dropped.⁶ Although this form (CES 6) continues to exhibit high serial correlation, all of the estimated parameters are large relative to their standard errors, except for the substitution parameter ρ , which appears as insignificant, a finding similar to the result of the Kmenta test reported in Sinai-Stokes (1972, p. 291, footnote 5). Also, the extremely high degree of collinearity between the estimated parameters is significantly reduced. In Sinai-Stokes (1972), the Kmenta approximation to the CES yielded –3.727 for the constant term or 0.02406 after taking antilogs. This figure is close to the value obtained in the nonlinear estimation of the CES, i.e., $\alpha = 0.03639$ in table 2, CES 6, and far from the unity assumed by BK. Thus, the nonlinear estimation suggests that the CES form would collapse to a CD.

⁵ If real balances are added to an unconstrained equation that has the CES 5 form, collinearity between parameters is still a problem and the results are insignificant.

⁶ This is a nontrivial assumption as the sequence of results in table 2 show. With a unitary scaling parameter, dividing both sides of a CES would produce the BK form (stochastic form, BK (1979, p. 443)) and permit the omission of a constant term from the ensuing regression. If the efficiency parameter were unequal to unity, then the specification of the model would be different and division of both sides of the equation would change the resulting estimates because of nonlinearities.

² See the BK functional and stochastic forms (II) (1979, p. 443). Assuming unity for the efficiency parameter simplified the estimation to be linear in the parameters and permitted OLS to be used in the second stage of the procedure.

³ Nadiri (1970), p. 1151.

⁴ Draper and Smith (1966), p. 273.

TABLE 2.—NONLINEAR ESTIMATION OF THE CES PRODUCTION FUNCTION
(WITH AND WITHOUT REAL MONEY BALANCES): 1929–1967

$$\text{CES 5 } Q = (\delta_1 K^\rho + \delta_2 L^\rho)^{1/\rho}$$

Correlation of Parameters

| | | | | |
|------------|---------|---------|---------|--------|
| δ_1 | 1.0000 | | | |
| δ_2 | 0.9963 | 1.0000 | | |
| v | 0.0623 | 0.1409 | 1.0000 | |
| ρ | -0.9978 | -0.9998 | -0.1237 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|-----------------|-------------|---------------------|
| δ_1 | 0.25673 | 0.5675 |
| δ_2 | 0.66770 | 0.5399 |
| v | 1.73350 | 33.38 |
| ρ | 0.04075 | 0.0380 |
| S.E.E. = 9.8278 | | D.W. = 0.8376 |

$$\text{CES 6 } Q = \alpha(\delta K^\rho + (1.0 - \delta)L^\rho)^{1/\rho}$$

Correlation of Parameters

| | | | | |
|----------|---------|---------|---------|--------|
| δ | 1.0000 | | | |
| α | 0.5054 | 1.0000 | | |
| v | -0.4024 | -0.9930 | 1.0000 | |
| ρ | 0.9448 | 0.2198 | -0.1091 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|-----------------|-------------|---------------------|
| α | 0.03639 | 3.176 |
| δ | 0.32153 | 2.679 |
| v | 1.73095 | 34.529 |
| ρ | 0.48399 | 0.443 |
| S.E.E. = 9.8053 | | D.W. = 0.8556 |

$$\text{CES 7 } Q = \alpha(\delta_1 K^\rho + \delta_2 L^\rho + (1.0 - \delta_1 - \delta_2)(M1/P)^\rho)^{1/\rho}$$

Correlation of Parameters

| | | | | | |
|------------|---------|---------|---------|---------|--------|
| δ_1 | 1.0000 | | | | |
| δ_2 | -0.9554 | 1.0000 | | | |
| α | 0.8404 | -0.8279 | 1.0000 | | |
| ρ | 0.6562 | -0.5812 | 0.1932 | 1.0000 | |
| v | -0.8157 | 0.7796 | -0.9948 | -0.1732 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|-----------------|-------------|---------------------|
| α | 0.08425 | 3.020 |
| δ_1 | 0.52018 | 8.776 |
| δ_2 | 0.33297 | 3.825 |
| ρ | 1.18481 | 2.572 |
| v | 1.61831 | 31.601 |
| S.E.E. = 8.2952 | | D.W. = 0.8588 |

$$\text{CES 8 } Q = \alpha e^{\gamma t}(\delta_1 K^\rho + \delta_2 L^\rho + (1 - \delta_1 - \delta_2)(M1/P)^\rho)^{1/\rho}$$

Correlation of Parameters

| | | | | | | |
|------------|---------|---------|---------|---------|---------|--------|
| δ_1 | 1.0000 | | | | | |
| δ_2 | -0.9670 | 1.0000 | | | | |
| α | 0.0710 | -0.0737 | 1.0000 | | | |
| ρ | 0.5739 | -0.5119 | 0.1179 | 1.0000 | | |
| v | 0.1256 | -0.1303 | -0.9761 | -0.0733 | 1.0000 | |
| λ | -0.5063 | 0.4977 | 0.7698 | -0.0070 | -0.8868 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|-----------------|-------------|---------------------|
| α | 0.08868 | 1.898 |
| δ_1 | 0.51588 | 7.253 |
| δ_2 | 0.33912 | 3.266 |
| ρ | 1.18366 | 2.484 |
| v | 1.60532 | 14.270 |
| γ | 0.0004 | 0.120 |
| S.E.E. = 8.4184 | | D.W. = 0.8510 |

Second order GLS form of CES 6

Correlation of Parameters

| | | | | | | |
|-------------|---------|---------|---------|--------|---------|--------|
| δ_1 | 1.0000 | | | | | |
| α | 0.9371 | 1.0000 | | | | |
| v | -0.9264 | -0.9995 | 1.0000 | | | |
| ρ | 0.1346 | 0.0798 | -0.0747 | 1.0000 | | |
| λ_1 | 0.2308 | 0.2555 | -0.2574 | 0.0542 | 1.0000 | |
| λ_2 | 0.1666 | 0.2058 | -0.2086 | 0.0119 | -0.5128 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|-----------------|-------------|---------------------|
| δ_1 | 0.27122 | 6.555 |
| α | 0.03648 | 2.691 |
| v | 1.72702 | 28.369 |
| ρ | 0.21237E-06 | 0.450E-04 |
| λ_1 | 0.95540 | 6.002 |
| λ_2 | -0.62013 | -3.959 |
| S.E.E. = 6.7980 | | D.W. = 1.4853 |

Second order GLS form of CES 7

Correlation of parameters

| | | | | | | | |
|-------------|---------|---------|---------|---------|---------|---------|--------|
| δ_1 | 1.0000 | | | | | | |
| δ_2 | -0.9581 | 1.0000 | | | | | |
| α | 0.6533 | -0.5863 | 1.0000 | | | | |
| ρ | 0.7722 | -0.7401 | 0.0825 | 1.0000 | | | |
| v | -0.5782 | 0.4833 | -0.9902 | -0.0149 | 1.0000 | | |
| λ_1 | -0.1249 | 0.1291 | 0.1900 | -0.2812 | -0.2185 | 1.0000 | |
| λ_2 | 0.0038 | 0.0110 | 0.1261 | -0.0857 | -0.1368 | -0.5365 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|-----------------|-------------|---------------------|
| δ_1 | 0.46333 | 5.837 |
| δ_2 | 0.41972 | 3.765 |
| α | 0.07655 | 2.705 |
| ρ | 0.76920 | 1.085 |
| v | 1.62733 | 27.427 |
| λ_1 | 0.98889 | 5.417 |
| λ_2 | -0.57380 | -3.327 |
| S.E.E. = 6.0888 | | D.W. = 1.709 |

Second order GLS form of CES 8

Correlation of parameters

| | | | | | | | | |
|-------------|---------|---------|---------|---------|---------|---------|--------|--------|
| δ_1 | 1.0000 | | | | | | | |
| δ_2 | -0.9896 | 1.0000 | | | | | | |
| α | -0.4633 | 0.4617 | 1.0000 | | | | | |
| ρ | 0.8637 | -0.8593 | -0.3333 | 1.0000 | | | | |
| v | 0.5988 | -0.5970 | -0.9840 | 0.4214 | 1.0000 | | | |
| λ_1 | -0.1469 | 0.1566 | 0.0861 | -0.0765 | -0.0765 | 1.0000 | | |
| γ | -0.7424 | 0.7222 | 0.8698 | -0.4595 | -0.9385 | -0.0032 | 1.0000 | |
| λ_2 | -0.2804 | 0.2775 | 0.3044 | -0.2082 | -0.3291 | -0.5489 | 0.3449 | 1.0000 |

| | Coefficient | <i>t</i> -statistic |
|------------------|-------------|---------------------|
| δ_1 | 0.32457 | 1.782 |
| δ_2 | 0.60237 | 2.478 |
| α | 0.18366 | 1.332 |
| ρ | 0.24472 | 0.191 |
| v | 1.40417 | 8.098 |
| λ_1 | 1.03552 | 5.739 |
| γ | 0.00741 | 1.287 |
| λ_2 | -0.57147 | -3.351 |
| S.E.E. = 6.00555 | | D.W. = 1.7692 |

Note: For a discussion of the data, see Sinai and Stokes (1972, p. 296). All equations were estimated using the Marquardt nonlinear algorithm GAUSHAUS, due to Meeter. See Draper and Smith (1966, p. 273) for a discussion. The *t*-statistics reported arise from confidence intervals for each parameter based on the linear hypothesis. δ = distribution parameter; α = efficiency (or scaling) parameter; ρ = substitution parameter; v = degree of returns to scale. S.E.E. = standard error of the estimate. Given that the basic model was in the form $Q_t = F_1(L, K, \dots)$ all GLS equations were estimated as $Q_t - \lambda_1 Q_{t-1} - \lambda_2 Q_{t-2} = F_1(\dots) - \lambda_1 F_1(\dots) - \lambda_2 F_1(\dots)$ where the GLS first and second order parameters (λ_1, λ_2) were estimated simultaneously with all other parameters.

When CES 6 is estimated using a second order GLS procedure, the parameters remain stable. As expected, the Durbin-Watson statistic improves to 1.49 and the substitution parameter ρ continues to be insignificant.⁷ These GLS CES results reinforce our prior finding that the CES is not the appropriate functional form. CES 7 and CES 8 show the effect of adding real balances and a time trend to the CES production function. When these models are corrected for serial correlation, ρ continues to be insignificant, the Durbin-Watson improves to 1.71 and 1.77, respectively, and there is a substantial reduction in the S.E.E. Our conclusion remains that the CD form, not CES, is the appropriate functional form and that real balances belong in the production function.

IV. Real Money Balances in the CES Production Function

BK claim that the addition of real money balances to the aggregate production function is a misspecification. Yet, their own results (BK, 1979, table 1, the CES estimations), reestimated by us with the BK two-stage method (not shown here), suggest that real M1 is highly significant (t -value = 2.5). In addition, the \bar{R}^2 is raised from 0.9946 in the two factor CES case to 0.9950 when real money balances (M1) are added. In our table 2, the addition of real M1 to the general form of the CES brings a substantial reduction in the S.E.E. (from 9.8053 to 8.2952), although with considerable serial correlation remaining.

How can the BK conclusions be reconciled with this standard way of evaluating evidence for the significance of an additional explanatory variable in an econometric model? According to BK and the earlier work by Ramsey and Zarembka (1971), the CES is the proper functional form. Clearly, real M1 is not an included irrelevant variable in the CES production functions that were estimated. The omitted variable case is a possibility, but entering a time trend as a proxy did not perturb the results obtained with real M1 (table 2, CES 7 and CES 8). By elimination, simultaneity remains as the candidate for specification error. But even a high degree of coefficient sensitivity to a consistent estimation method might not invalidate the significance for the real cash balances coefficient.

V. Omitted Variables, Incorrect Functional Form, or Simultaneity?

BK report a Group A specification error (omitted variable, incorrect functional form, or simultaneity)

⁷ Unlike our previous work where we performed a second order GLS using a two-step procedure, in this formulation the GLS parameters λ_1 and λ_2 were estimated simultaneously with the other parameters in the model.

for the three-factor CES containing either real M1, M2 or M3, but they do not identify which Group A error. Since the two-factor CES passes the Ramsey test for Group A errors, the problem is not the functional form. And, if the two-factor CES were free from Group A errors, adding real money balances would not create a case of omitted variables. However, the addition of the money balances variable could have brought about a problem of simultaneous equations bias.

We have always argued that simultaneity was a potential problem. In previous work, we stated that a full simultaneous equations model should be specified and estimated to obtain consistent estimates of the real balances coefficient.⁸ But even if simultaneity were present, it would only mean that the estimator for the regression coefficient of real balances was inconsistent, and not necessarily that real money balances are an inappropriate input in the production function. Indeed, in several studies (Khan and Kouri, 1975; Butterfield, 1975; Short, 1979) where a simultaneous equations model has been specified, the role of real money balances in the production function has been supported.⁹

VI. Real Money Balances in Production: Additional Support with Micro Data

In our original work, we deliberately tried to avoid becoming involved in "difficult problems of specification and estimation when our purpose was simply to examine the potential significance of real balances in the production function."¹⁰ Thus, we concentrated on tests with aggregate annual data even though the rationale for the role of money balances in production was at the micro level. However, since Nadiri (1969) and others have found stable demand equations for real balances by firms assuming money as a productive input, we have attempted to verify our original findings with more disaggregated data. In table 3, some additional findings are presented, using quarterly data for the U.S. nonfinancial corporate sector over the period 1953:1 to 1977:3.

⁸ Sinai and Stokes (1972, p. 292; 1975, p. 250; 1977, p. 373).

⁹ In the most recent study of this type on real money balances and production (Short, 1979), two production function specifications, CD and translog, were tried and the system, which included three behavioral decision equations, was estimated using both OLS and two-stage least squares (2SLS). Short found that real balances were a significant input in the production function, even after correcting for simultaneity. Because the interaction terms in the translog specification were not significant, Short was unable to reject the CD specification. The Khan-Kouri (1975) and Butterfield (1975) results have been cited by us in a similar context in other replies, Sinai-Stokes (1975, 1977).

¹⁰ Sinai and Stokes (1972, p. 291).

TABLE 3.—COBB-DOUGLAS PRODUCTION FUNCTION (WITH AND WITHOUT REAL MONEY BALANCES): NONFINANCIAL CORPORATE SECTOR, 1953:1 TO 1977:3

$$\ln Q = \ln A + \alpha \ln L + \beta \ln K + \delta \ln (M/P) + \lambda t + u$$

(all equations estimated using second order GLS)

| | Equation Number | | | | |
|---------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| | 9 | 10 | 11(M1) | 12(M2) | 13(FA) |
| $\ln A$ | -2.152 (13.34) | 0.4638 (1.57) | 0.4335 (1.70) | 0.3488 (1.48) | 0.3395 (1.40) |
| α | 1.5735 (19.76) | 0.8502 (10.39) | 0.8656 (12.28) | 0.8843 (13.85) | 0.8071 (11.73) |
| β | 0.3333 (8.82) | 0.3319 (16.80) | 0.2748 (10.51) | 0.2721 (11.04) | 0.3217 (19.82) |
| δ | | | 0.0869 (2.88) | 0.0963 (3.06) | 0.0822 (2.46) |
| λ | | 0.0040 (9.53) | 0.0044 (11.46) | 0.0040 (12.01) | 0.0040 (11.61) |
| $\alpha + \beta + \delta$ | 1.9068 | 1.1821 | 1.2273 | 1.2527 | 1.2110 |
| \bar{R}^2 | 0.9676 | 0.9934 | 0.9951 | 0.9962 | 0.9961 |
| D.W. | 1.971 | 2.136 | 2.026 | 1.999 | 2.038 |
| S.E.E. | 0.0601 | 0.02651 | 0.02264 | 0.01999 | 0.02027 |

Data sources: Federal Reserve Flow-of-Funds and Data Resources.

Note: Variable definitions are Q = real gross domestic product of the nonfinancial corporate sector; L = employment in the nonfinancial corporate sector; K = gross capital stock of plant and equipment times the FRB capacity utilization index; $M1$ = real balances (M1) held by nonfinancial corporations; $M2$ = real balances (M2) held by nonfinancial corporations; FA = real financial assets held by nonfinancial corporations (= $M1$ + time deposits + U.S. Government securities + State and local government securities + commercial paper + security RP's); P = implicit deflator for gross domestic product.

The standard errors of estimate are substantially reduced by the inclusion of real balances, defined alternatively as M1, M2 or financial assets. The coefficients remain highly significant even in equations with a time trend. The returns to scale drop sharply toward unity once a time trend and measures for real money balances are included. Employing the methodology suggested by Pierce (1977) and used recently in another context by Frenkel (1977), we also tested our new data for the direction of causality between the inputs and output in the sense of Granger (1969), finding results consistent with the hypothesis that changes in real balances cause changes in real output.¹¹

¹¹ Our testing does not rule out what Pierce (1977) has called instantaneous causality, only feedback. The first step in our procedure involved estimation of Box-Jenkins (1976) ARIMA prewhitening filters for the output series ($\ln Q$) and all possible inputs ($\ln K$, $\ln L$, $\ln M1$, $\ln M2$, $\ln FA$). The adequacy of the chosen filter was verified using the Ljung-Box (1976) modified Q -statistic. Our results indicate that only L and K are related contemporaneously to Q (cross correlations of 0.79 and 0.78 between the prewhitened series), while for $M1$ and $M2$ the relationship is both in the contemporaneous period (0.37 and 0.24) and at a three quarter lag (0.22 and 0.32, respectively). Although the Pierce test is known to be biased against finding a significant relationship between two series (Stokes and Neuburger, 1979) our evidence is consistent with the hypothesis that real balances are exogenous variables in the production function. Only with the alternative parameterization of the real balances term as FA is there weak evidence of feedback from Q to FA (0.22 for lag -1 and

VII. Conclusion

In summary, the BK work fails to controvert the notion that real money balances are a productive input, mistakenly omitted from the production function. Using a more powerful specification error test suggested by Theil (1971), we found strong evidence that the BK "ideal" two-factor CES model, without money balances, had both severe heteroscedasticity and serial correlation problems. In CES production functions containing real balances, estimated by BK and us, the coefficient of the M1 term was always large relative to its standard error, actually providing support for the argument that real money balances are an input to the aggregate production function. We noted that the BK results are highly sensitive to the assumption of unity for the scale coefficient in the CES function and to the particular two-stage linear estimation procedure used. Non-linear estimation of the CES and relaxation of the unitary constraint resulted in the finding that real balances are a significant input in the production function. The BK results indicate that the most likely specification error is simultaneity, just as we have previously argued, the effects of which have been examined by Khan-Kouri (1975), Butterfield (1975) and Short (1979), with results that support real balances as a factor input.

0.28 for lag -6). The underlying data for these results can be obtained from the authors.

We also reported additional evidence for the role of real balances in production using a disaggregated set of data for U.S. nonfinancial corporations and a CD specification with nonconstant returns to scale. Using time-series methods, we found no evidence of feedback from output to real money balances, measured either as M1 or M2. We conclude that although further work in this area may yet discover more appropriate inputs for the traditional production function, the evidence to date suggests that real money balances will have to be included.

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