THE PURE THEORY OF INTERNATIONAL TRADE

By Robert Alexander Mundell*

The English classical model of foreign trade is the source of many propositions which form the body of international trade theory today. Despite attacks on other branches of classical theory it still survives as a basic tool of analysis. Its survival can be attributed to its applicability to leading policy issues in the country in which it originated, and to the power of its methodology: it was logically immune to the criticisms of general equilibrium and macroeconomic analysis.

The classical economists were content to establish the direction in which the terms of trade move as a result of such disturbances as dis-hoarding, tariff adjustments, devaluation, income transfers and productivity changes. Nowadays more refined methods make it possible to derive more implications from the model, implicit in their analysis, and to ascertain the quantitative extent of the change in the terms of trade. The purpose of this paper is to derive and summarize these results.

Specifically, I shall construct an international trade model owing its origin to the classical school, and apply it to determine the exact effects on international equilibrium of unilateral transfers, productivity changes, export and import taxes, and production and consumption taxes. Many of the conclusions are already known, but it is believed that the methods employed will help to simplify the techniques used in this branch of international trade theory, and that the results established will provide a convenient survey of the subject. The first part of the analysis will be concerned with the implications of the two-country two-commodity model usually employed by the classical economists. In the final section an attempt is made to determine the validity of the results when there are many countries.

I. The Free Trade Model

Assume that there are two countries, A and B, in full employment, producing two commodities—X, which is exported by A, and Y, which is exported by B. Let capital letters denote production, small letters consumption, and subscripts, countries. Let $T$ represent the capital

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exports (lending) of country A expressed in terms of \( X \); let \( P \) denote the terms of trade, the price of \( Y \) in terms of \( X \); and let \( D \) represent domestic expenditure.

The system can then be described by the following equations:

1. \( D_a = x_a + P y_a = X_a + PY_a - T \).
   Domestic expenditure in A (in terms of \( X \)) equals national income minus net capital exports.

2. \( D_b = \frac{x_b}{P} + y_b = \frac{X_b}{P} + Y_b + \frac{T}{P} \).
   Domestic expenditure in B (in terms of \( Y \)) equals national income plus net capital imports.

3. \( y_a = y_a(D_a, P) \).
   The demand for \( Y \) in A depends on domestic expenditure and the terms of trade.

4. \( x_b = x_b(D_b, 1/P) \).
   The demand for \( X \) in B depends on domestic expenditure and the terms of trade.

5, 6. \( X_a = X_a(1/P) \); \( Y_a = Y_a(P) \).
   The production of \( X \) and \( Y \) in A depends on the terms of trade.

7, 8. \( X_b = X_b(1/P) \); \( Y_b = Y_b(P) \).
   The production of \( X \) and \( Y \) in B depends on the terms of trade.

9. \( T = x_b - X_b - P(y_a - Y_a) \).
   The net capital exports of country A equal the balance of trade of country A.

Variations in domestic expenditure in each country are assumed to depend on changes in policy. In the free-trade case the system is completed by the following equations:

\( D_a = D_a(T) \); and \( D_b = D_b(T/P) \).

We then have eleven independent equations in the twelve unknowns: \( x_a, x_b, y_a, y_b, X_a, X_b, Y_a, Y_b, D_a, D_b, P \) and \( T \), so there is one degree of freedom. Knowing the rate at which A is lending to B (i.e., \( T \)), we can solve for the equilibrium terms of trade \( (P) \); or, assuming that the terms of trade are fixed, we can find the rate of lending which will establish equilibrium.\(^1\)

There are other, equivalent ways of expressing the same system.

\(^1\) The system can be represented in one diagram by the Edgeworth-Bowley box diagram if production is fixed, and by the technique introduced by Meade [13] if production is variable. The diagrams in the text provide an alternative proof of some of the propositions; but if the reader prefers to do so he can follow the argument without reference to the diagrams.
Equations (1) and (2) could be replaced by conditions stating that world production and world consumption of each good must be equal—these alternatives imply each other when combined with equation (9) expressing balance-of-payments equilibrium. Equations (3) and (4), the demand functions for the good which is imported in each country, could be replaced by the demand functions for the good which is exported since all income is spent—the part of domestic expenditure which is not spent on one good must be spent on the other good.

It will be convenient to define an import demand function for each country. The demand for imports is the difference between the quantities of the imported good demanded and supplied, i.e., $I_a = y_a - Y_a$, and $I_b = x_b - X_b$, where $I_a$ and $I_b$ are, respectively, the demands for imports in A and B. Then since the demand and supply functions depend only on domestic expenditure and the terms of trade, the import demand functions must also depend on these variables. Thus we have two more equations and two more unknowns:

$$I_a = I_a(D_a, P) \quad \text{and} \quad I_b = I_b(D_b, 1/P).$$

If we now substitute (10) and (11) in the balance-of-payments equation (9) we obtain

$$T = I_b(D_b, 1/P) - PI_a(D_a, P).$$

The task of the following analysis is to introduce into these equations various policy parameters, and to show how the equilibrium values of the variables are affected by changes in these policies. It is first convenient, however, to outline the procedure by which the effect of these changes may most simply be obtained.

II. Procedure

The purpose of comparative-statics analysis is to compare two positions of equilibrium distinguished from each other by a shift in some parameter. In this paper we call the parameter shift a “policy change.” A policy change disturbs the initial equilibrium by causing an excess demand for one of the commodities which must be eliminated by an adjustment in some other variable at the new equilibrium. The adjusting or “equilibrating” variable may be another policy change, a reversal of the original policy change, or a process of adjustment which is sufficiently traditional—hence predictable—to be called “automatic.”

In classical theory the adjustment mechanism was automatic. A policy change disturbed balance-of-payments equilibrium, induced a gold flow and, through changes in relative price levels, a change in the terms of trade. Today this mechanism is not so automatic, i.e., central bank and government reaction to disequilibrium in the balance of payments is less predictable. Besides the traditional inflation-deflation
method of the gold standard a disequilibrium may be corrected by borrowing (in the short run), trade controls, tax changes, technological change (in the long run), or exchange rate adjustment. Most of these methods have been used by one country or another since the breakdown of the gold standard system to resolve balance-of-payments crises.

Because of this change in institutional response to disequilibrium any analysis of policy changes must be taxonomic. Questions like: “Do tariffs improve the balance of trade?” cannot be given an unequivocal reply—the answer depends on the other policies followed by the government. A tariff disturbs the initial equilibrium and therefore requires, for a new equilibrium to be reached, a change in some other policy; it may involve changes in any or all of the policies listed above.

But exploring all conceivable policy alternatives would be tedious and unrewarding; limits have to be imposed somewhere. For that reason I shall tentatively assume that the classical mechanism is operative, that the terms of trade “automatically” adjust to correct disequilibrium.2 The first part of the comparative-statics analysis will therefore determine the effect of policy changes on the terms of trade. It will be shown later how the results can be manipulated to demonstrate the working of other mechanisms of adjustment.

The simplest way to derive the effect of a policy change on the terms of trade is to differentiate the balance-of-payments equation (12) with respect to the change in policy, and to substitute in the result the conditions necessary to satisfy the other conditions of equilibrium. A more intuitive way of getting the criterion, however, is to employ a device implicit in all comparative-statics analysis. This is to compute the excess demand caused by the policy change on the assumption that the adjusting variable (the terms of trade) is constant; and to equate this excess demand to the excess supply created by the actual change in the terms of trade. If, for example, we wish to find the criterion for the effects of a tax on the terms of trade, we first determine the excess demand caused by the tax at constant terms of trade and translate the coefficient of the tax change into the appropriate income or price elasticity; we then compute the excess supply of the same good created by a change in the terms of trade, translating its coefficient into the relevant elasticities. By equating the excess demand and the excess supply the criterion is established.3

2 The terms of trade may be assumed to change by means of exchange rate or price level adjustments. If e is the price of a unit of B’s currency in terms of A’s currency, and \( P_a \) and \( P_b \) are the factor cost prices of exports in A and B, the terms of trade are \( T = e \cdot P_b / P_a \) in the free trade case. From a position of initial equilibrium an increase in e (devaluation by country A) would ultimately cause an offsetting change in \( P_b / P_a \) leaving the terms of trade and the balance of payments unaltered.

3 This procedure, which may be called the “method of comparative statics,” can be illus-
One further point must be investigated. The adjustment mechanism implies a type of dynamic behavior and thus a condition of dynamic stability. If the system is unstable it would not tend to approach the new equilibrium given by the comparative-statics analysis, and there would be little point in pursuing the comparative-statics analysis. On the other hand if the system is stable, a useful clue may be obtained from the stability conditions about the sign of the coefficient of the adjusting variable, the terms of trade.\(^4\) The first step, then, is to examine the conditions of dynamic stability.

4 By the correspondence principle [22, Ch. 9]. Thus in the first example cited in the pre-
III. Stability Conditions

An equilibrium is stable if a small displacement is followed by a return to equilibrium. I assume that the above system is stable if a displacement of the terms of trade from equilibrium sets in motion forces inducing a return to that equilibrium. Now a disequilibrium in the classical system induces a gold flow and a deterioration of the terms of trade of the deficit country: the system is therefore stable only if a fall in the terms of trade of the deficit country causes an improvement in its balance of payments. To find the stability conditions we need to compute the excess supply caused by a change in the terms of trade. This excess supply will simultaneously establish the coefficient of a change in the terms of trade for use in the comparative-statics analysis.

With no lending, and expenditure in each country constant, the balance of payments \((B)\) of country A in terms of home goods can be written as follows:

\[
B = I_b \left(1/P\right) - P \left(1/I_a\right).
\]

In equilibrium this must be zero. Now choose commodity units so that \(P\) is initially equal to unity; then at equilibrium the volume of B's imports equals the volume of A's imports so that we can write \(I_b = I = I_a\) initially. Differentiating (13) we get

\[
\frac{dB}{dP} = - \frac{PdI_b}{d(1/P)} - \frac{PdI_a}{d(1/P)} - I_a = I \left( - \frac{P}{I_b} \frac{dI_b}{d(1/P)} - \frac{P}{I_a} \frac{dI_a}{dP} - 1 \right).
\]

The first two terms in the bracket are, respectively, the elasticities of demand for imports in A and B; write these terms as \(\eta_a\) and \(\eta_b\). For stability a fall in A's terms of trade must improve A's balance of payments so that the system is stable or unstable depending on whether:

\[
\frac{dB}{dP} = I(\eta_a + \eta_b - 1) \geq 0.
\]

In words, the system is stable depending on whether the sum of the elasticities of demand for imports is greater or less than unity.\(^5\) This is,

\(^5\) The dynamic behavior of the system may be approximated by the following differential equation:

\[
\frac{dP}{dt} = k \left[ P I_a(P) - I_b(1/P) \right]
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of course, the familiar Marshallian condition.\(^6\) (See Figure 1.)

The stability condition can be expressed in terms of one good only.
To see this, recall equation (1) which expresses the equality of income (plus borrowing) and expenditure in country \(A\). With lending zero this equation can be written:

\[ X_a - x_a = P(y_a - Y_a) = P I_a. \]

i.e., offers of exports equal the value of imports demanded. Substituting in (13) and making a similar substitution for \(I_b\), we can write the balance of payments (with no lending) as follows:

\[ B = (x_b - X_b) - (X_a - x_a). \]

Differentiating and rearranging terms we get:

\[ \frac{dB}{dP} = \left( \frac{d(x_a + x_b)}{dP} \frac{P}{(x_a + x_b)} - \frac{d(X_a + X_b)}{dP} \frac{P}{(X_a + X_b)} \right) X \]

where \(X\) is world production and consumption at equilibrium, and the arguments in the bracket are, respectively, the world elasticity of demand for \(X\) and the world elasticity of supply of \(X\):

\[ \eta_x = -\frac{d(x_a + x_b)}{d(1/P)} \frac{1}{(x_a + x_b)} \]

\[ \epsilon_x = \frac{d(X_a + X_b)}{d(1/P)} \frac{1}{(X_a + X_b)} \]

which states that the speed of the change in the terms of trade is proportional to the discrepancy between foreign exchange payments and receipts. Expanding (1) in a Taylor series, omitting nonlinear terms, and choosing time units to make \(k=1\), we obtain

\[ \frac{dP}{dt} = (1 - \eta_a - \eta_b)(P - P^0) \]

where \(P^0\) is the terms of trade at equilibrium. Equation (2) has a solution

\[ P = P^0 + Ae^{-t(\eta_a + \eta_b - 1)}. \]

The equilibrium point is stable only if \(P\) eventually approaches \(P^0\); this can only be the case if the other term in (3) disappears, i.e., if \(\eta_a + \eta_b - 1 > 0\).

\(^6\) Marshall's dynamic postulates differ from those described in the text and in the preceding footnote. The latter assumes that the budget equations in each country are instantaneously satisfied (each country is always at a point on its offer curve) but that markets are not necessarily cleared. Marshall's postulates are based on adjustments of offers toward the budget equations (offer curves) \([11]\). This difference corresponds roughly to the distinction made between instantaneous and lagged adjustments analyzed in Arrow and Hurwicz \([1]\). Marshall's adjustment process, which is rationalized by the varying profitability of export industries, admits the possibility of complex roots and therefore an oscillatory path to equilibrium. See Samuelson \([22, pp. 266-68]\).

This discussion refers to the stability of an equilibrium rather than to the stability of a system. John Stuart Mill recognized the possibility of multiple equilibria without reference to stability \([17, pp. 154-63]\), but his treatment was faulty, Marshall, in 1879, was aware \([11, pp. 24-25]\) that a point of unstable equilibrium must be flanked by points of stable equilibria, that the number of equilibria must be odd, and that (therefore) if an equilibrium were unique it would be stable.
The offer curves $OA$ and $OB$ intersect at initial equilibrium $Q$. A change in the terms of trade in the proportion $TQ/QM$ in favor of $A$ creates an excess demand for $B$'s good and a deficit in $A$'s balance of payments equal to $RW$ (in terms of $Y$). The situation is therefore stable: Country $A$ loses and country $B$ gains gold, causing deflation in $A$ and inflation in $B$ until the gap $RW$ is closed at the original equilibrium $Q$.

The stability conditions are derived as follows: Define the elasticities of demand for imports in each country,

\[ \eta_a = \frac{RQ}{QM} / \frac{TQ}{QM} \quad \text{and} \quad \eta_b = \frac{TW}{QM} / \frac{TQ}{QM} \]

and the balance of payments deficit in $A$ in terms of foreign goods,

\[ \frac{dB}{P} = RW. \]

Then:

\[ \frac{dB}{P} = RW = QM \cdot \frac{TQ}{QM} \cdot \frac{RW}{TQ} = QM \cdot \frac{TQ}{QM} \cdot \frac{RQ + TW - TQ}{TQ} = QM \cdot \frac{TQ}{QM} \left[ \frac{RQ}{QM} \cdot \frac{QM}{TQ} + \frac{TW}{QM} \cdot \frac{QM}{TQ} - 1 \right] = (\text{by definition}) \ I \ \frac{dP}{P} \ (\eta_a + \eta_b - 1). \]

Hence:

\[ \frac{dB}{dP} = X(\eta_a + \epsilon_a), \]

where units are chosen so that $P$ is, at equilibrium, equal to unity. By
similar reasoning

\[ \text{from the balance of payments equation, } \frac{B}{P} = (Y_b - y_b) - (y_a - Y_a) \]

we find that:

\[ \frac{dB}{dP} = Y(\eta_y + \epsilon_y), \]

where \( \eta_y \) and \( \epsilon_y \) are the elasticities of world demand for, and supply of, \( Y \).

The elasticities of demand for \( X \) and \( Y \) are defined to be positive provided that these goods are not Giffen goods; and the elasticities of supply of \( X \) and \( Y \) are defined to be positive provided that opportunity costs are not decreasing. It may then be seen that the system is necessarily stable if neither good is a Giffen good and opportunity costs are not decreasing. But even if the goods are Giffen goods, positive supply elasticities may yet make the system stable.

In the remainder of this paper I assume that the stability condition is satisfied, an assumption which, as the foregoing remarks suggest, does not appear very restrictive.

IV. Unilateral Payments

The first policy change we shall consider is a unilateral payment from one country to the other. This involves two parts: a financial transfer and a real transfer. The financial transfer refers to the accumulation and liquidation of debt on the part of individuals or governments in each country, while the real transfer refers to the induced movement of goods. Assume that A is the transferring country.

In the case of a private flow of capital (ignoring interest payments) lenders in A buy the debt of borrowers in B, the former financing the purchase out of an excess of saving over investment, the latter disposing of the proceeds by an excess of investment over saving. Because of the identity of income-less-lending and expenditure [equations (1) and (2)] the excess of investment over saving in B, and saving over investment in A, must each equal the transfer.

In the case of intergovernmental transfers such as reparations payments or foreign aid, the government in A (the paying country) grants credits to B, the former financing the credits by means of, say, an in-

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7 Mosak derived stability conditions in terms of one good only [18, Ch. 4]. See also Johnson [6, p. 98].

When trade is not initially in balance a slight adjustment is required in the stability condition. See Hirschman [5].

8 Marshall’s judgment is probably too strong: “...it is not inconceivable, but it is absolutely impossible” [12, p. 354].
come tax, the latter disposing of the proceeds by means of, say, an income subsidy. Again, because of the identity of income-less-lending and expenditure, the budget surplus in A and the budget deficit in B are each equal to the transfer.

Whatever the type of transfer and however it is financed and disposed of, domestic expenditure in A is reduced, and in B is increased, by the amount of the transfer. These changes in expenditure induce changes in demand which, at constant terms of trade, create disequilibrium in the balance of payments. The transfer problem may then be posed as the problem of determining the direction and extent of the change in the terms of trade required to eliminate the balance-of-payments disequilibrium.

The Terms of Trade. To find the effects of a transfer on the terms of trade we first determine the excess demand created by the expenditure changes at constant terms of trade. This can be done in terms of either good since an excess demand for one good implies an excess supply of the other good.

The reduction in domestic expenditure in A decreases the demand for $Y$ in A at constant terms of trade by:

$$P \frac{\partial y_a}{\partial D_a} dD_a = m_a dD_a$$

where $m_a$ is the marginal propensity to spend on imports in A. The increase in domestic expenditure in B increases the demand for $Y$ in B by:

$$\frac{\partial y_b}{\partial D_b} dD_b = c_b dD_b$$

where $c_b$ is the marginal propensity to spend on home goods in B. The excess demand at constant terms of trade is therefore the sum of these changes or:

$$m_a dD_a + c_b dD_b = (c_b - m_a) dT,$$

noting that the changes in expenditure in each country are equal to the change in lending, i.e., $-dD_a = dD_b = dT$. Now expenditure in each country is divided between home goods and imports so that the sum of the marginal propensities to spend on home goods and imports is unity; thus $c_b + m_b = 1 = c_a + m_a$. We can now make use of this result to translate the above criterion into a number of equivalent forms. The

\[\text{Differentiation of } D_a = x_a + Py_a \text{ and } D_b = x_b/P + y_b \text{ with respect to } D_a \text{ and } D_b \text{ yields:}
\]

$$1 = \frac{\partial x_a}{\partial D_a} + P \frac{\partial y_a}{\partial D_a} = c_a + m_a, \text{ and}

$$1 = \frac{1}{P} \frac{\partial x_b}{\partial D_b} + \frac{\partial x_b}{\partial D_b} = m_a + c_a.$$
most convenient for our purposes is the familiar one:

\[(1 - m_a - m_b) dT,\]

which states that transfer creates an excess demand for, or excess supply of, the good of the transferring country depending on whether the sum of the marginal propensities to spend on imports is greater or less than unity. Only in the special case where the receiving country increases its consumption of the two goods in the same proportion that the paying country does without them \((1 - m_a - m_b = 0)\) will no change in the terms of trade be required. If \(m_a + m_b > 1\) the receiving country experiences a deficit; if \(m_a + m_b < 1\), the paying country suffers a deficit. (See Figure 2.)

To correct the disequilibrium equal to \((1 - m_a - m_b) dT\) a change in the terms of trade is required. But we already know from the stability condition \((14)\) that a change in the terms of trade causes an excess supply of B’s good (or an excess demand for A’s good, or improves A’s balance, or worsens B’s balance) by an amount equal to:

\[(\eta_a + \eta_b - 1) IdP.\]

The excess demand for B’s good at constant terms of trade must, at the new equilibrium, be equal to the excess supply of B’s good caused by the actual change in the terms of trade. Equating of \((16)\) and \((17)\) therefore provides the general criterion for the change in the terms of trade:

\[\frac{dP}{dT} = \frac{1 - m_a - m_b}{1(\eta_a + \eta_b - 1)}.\]

It may be seen that the higher are the price elasticities of demand for imports the smaller will be the change in the terms of trade (a small change relieves a large excess demand). In the limiting case where one of the elasticities is infinite, no change in the terms of trade is required. Similarly, the closer to unity is the sum of the marginal propensities to import, the smaller is the excess demand to be eliminated by a change in the terms of trade, and so the smaller is the actual change in the terms of trade.\(^{10}\)

\(^{10}\) To obtain the criterion directly, differentiate the balance of payments equation:

\[T = I_s(D_h, 1/P) - P I_a(D_a, P)\]

with respect to \(T\). This yields:

\[1 - m_b \frac{dD_b}{dT} + m_a \frac{dD_a}{dT} = \frac{\partial I_s}{\partial (1/P)} \frac{d(1/P)}{dT} - P \frac{\partial I_a}{\partial P} \frac{dP}{dT} - I_a \frac{dP}{dT}.\]

Expenditure changes are equal in absolute value to the transfer so

\[\frac{dD_b}{dT} = - \frac{dD_a}{dT} = 1.\]

Then by forming elasticities from the terms on the right, and taking \(P\) initially equal to unity, we get \((18)\).

For a sample of recent literature on the transfer problem see [18, Ch. 4] [15] [7] [23]; and for a survey of earlier literature see Viner [24, pp. 290-377].
The pretransfer offer curves (not drawn) are assumed to intersect initially at $Q'$. Assume that country A makes a payment equal to $OH$ of $X$ to country B. Then at constant terms of trade expenditure in A is reduced and in B is increased by $OH$ of $X$ or $ST$ of $Y$. These changes in expenditure induce A to buy $UR$ less of $Y$ and B to buy $UW$ more of $Y$, creating an excess demand for $Y$ and a deficit in A’s balance of payments equal to $RW$. To eliminate this excess demand a deterioration in A’s terms of trade is required until a new equilibrium such as $Q$ is achieved. The new equilibrium $Q$ is determined by the intersection of the new offer curves $HA$ and $HB$ which originate from the new endowment-position $H$.

The criterion for the change in the terms of trade following transfer may be derived as follows: Define the marginal propensities to import in each country,

$$m_a = \frac{UR}{ST} \quad \text{and} \quad m_b = 1 - c_b = 1 - \frac{UW}{ST} = \frac{SU - TW}{ST},$$

and the transfer

$$\frac{dT}{P} = \frac{OH}{OH/ST} = ST \text{ in terms of } Y.$$

Then the deficit in the balance of payments of country A due to the transfer at constant prices (in terms of $Y$) is:

$$\frac{dB}{P} = RW = \left[ \frac{UW}{ST} - \frac{UR}{ST} \right] ST = \left[ 1 - \frac{SU - TW}{ST} - \frac{UR}{ST} \right] \frac{ST}{OH} OH = (\text{by definition}) (1 - m_a - m_b) \frac{dT}{P}.$$

On the other hand we also have, from Figure 1,

$$\frac{dB}{P} = QM \cdot \frac{TQ}{QM} \left[ \frac{RQ}{QM} \cdot \frac{QM}{TQ} - \frac{TW}{QM} \cdot \frac{QM}{TQ} + 1 \right] = \frac{dP}{P} (\gamma_a + \gamma_b - 1).$$

$$\therefore \frac{dP}{dT} = \frac{1 - m_a - m_b}{I(\gamma_a + \gamma_b - 1)}.$$
Real Income. How is real income affected by a grant or gift from one country to the other? In the reparations discussions of the interwar period the view was widely held that the terms of trade of the paying country must fall, thus imposing an additional burden. The change in real income implicit within the change in the terms of trade was called the "transfer burden."

The change in real income due to transfer is composed of two effects—the direct effect of the change in expenditure, and the income effect implicit within the change in the terms of trade. Thus the real income of the receiving country improves by more or less than the transfer itself depending on whether the terms of trade improve or worsen. For small changes we can approximate the income effect of a change in the terms of trade by the change in cost of the initial volume of imports, i.e., by $IdP$. The change in real income as a result of transfer is therefore approximately:

$$\frac{dU_b}{dT} = - \frac{dU_a}{dT} = 1 + I \frac{dP}{dT}$$

where $U_a$ and $U_b$ are, respectively, the real incomes of A and B. Now substituting for $dP/dT$ from (16) we obtain an approximate quantitative measure of the change in real income evaluated at pretransfer prices:

$$\frac{dU_b}{dT} = - \frac{dU_a}{dT} = 1 + \frac{1 - m_a - m_b}{(\eta_a + \eta_b - 1)}.$$

It will now be convenient to introduce a relationship between price elasticities and income propensities based on Slutsky’s separation of price effects into income and substitution effects. A price elasticity of demand can always be written as the sum of a compensated (pure substitution) price elasticity and an income propensity. We can therefore write the price elasticity of demand for imports as the sum of the compensated elasticity of demand for imports and the marginal propensity to spend on imports. If the primes denote the compensated elasticities we have $\eta_a = \eta_a' + m_a$ and $\eta_b = \eta_b' + m_b$. Since pure substitution effects

Consider any demand function of the form $I = I(D, P)$ and differentiate with respect to $P$. This yields:

$$\frac{P}{I} \frac{dI}{dP} = P \frac{\partial I}{\partial D} \frac{dD}{dP} + \frac{P}{I} \frac{\partial I}{\partial D}$$

after multiplying by $P/I$. Now $-P/I \partial I/\partial P$ is the (money-income constant) elasticity of demand, $\eta$, and $P \partial I/\partial D$ is the marginal propensity to spend, $m$. Thus:

$$\frac{P}{I} \frac{dI}{dP} = m \frac{dD}{dP} - \eta.$$

A change in price can be associated with a change in real income approximately equal to the change in cost of the initial amount bought, $IdP$. If expenditure is adjusted to compensate for
are, in the two-good case, always positive the ordinary elasticity of demand for imports is always larger than the marginal propensity to import by an amount which depends on the size of substitution effects. (See Figure 3.)

Using this relationship we can manipulate (20) to get:

\[
\frac{dU_b}{dT} = -\frac{dU_a}{dT} = \frac{\eta_a - m_a + \eta_b - m_b}{\eta_a + \eta_b - 1} = \frac{\eta_a' + \eta_b'}{\eta_a + \eta_b - 1}.
\]

From this criterion it can be seen that the real income of the receiving country can only decline, as a result of transfer, if the system is unstable. Assuming stability, the higher are \(m_a\) and \(m_b\) (the marginal propensities to import) the smaller is the increase in real income of the receiving country, since income effects enter only in the denominator of (21).

V. Productivity Changes

Assume that the country experiencing the productivity increase is completely specialized and that expenditure increases by the full amount of the increase in output. We shall first determine the effects of a change in productivity on the terms of trade, and then its influence on real income.

The Terms of Trade. To determine the effect of a change in productivity on the terms of trade we first determine the excess demand created for one of the goods on the assumption that the terms of trade this change in real income, i.e., if \(dD = IdP\), then \(-P/I dI/dP\) becomes the compensated elasticity of demand, \(\eta'\), and we get

\[\eta = \eta'.\]

If indifference curves are convex all substitution effects are positive, so \(\eta' > 0\). From this it follows that the elasticity of demand for imports is always greater than the marginal propensity to import. \textit{A fortiori} the sum of the elasticities of demand for imports is greater than the sum of the marginal propensities to import so that if \(m_a + m_b\) is equal to, or exceeds, unity the exchange market is necessarily stable. Alternatively, an unstable exchange market implies that the sum of the marginal propensities to import is less than unity. J. E. Meade has made extensive use \[14\] of this relation; his stability condition is split into income and substitution effects and, in our notation, is \(\eta_a' + \eta_b' + m_a + m_b - 1\).

\[\text{Leontief produced an example} \[9\] \text{consistent with convex indifference curves, where the change in the terms of trade in favor of the paying country is so great that the real income of the latter improves as a result of the transfer. Equation (21) proves that this cannot happen unless the system is unstable. The identification of this "Leontief Effect" with instability is due to Samuelson} \[22, p. 29\].\]

\[\text{Transfer analysis has many applications in economic theory. It applies to any redistribution of income between sectors, individuals or groups within a country. In the Keynesian problem of income redistribution a gift or tax-cum-subsidy from the rich to the poor increases or decreases effective demand depending on whether the marginal propensity to spend (MPS) of the rich is less or greater than that of the poor. In public finance theory an increase in government spending financed by new taxes stimulates effective demand if the MPS of the government is greater than that of the public. And in monetary theory a fall in the price level stimulates effective demand if the MPS of creditors is greater than that of debtors (including governments and central banks).}\]
Initial equilibrium is at $Q$ on A's offer curve $OA$. Suppose that the terms of trade change in the proportion $TQ/QM$, and that at the new terms of trade A trades at the point $Q'$. The income effect of this change in the terms of trade can be approximated by $OD$ in terms of $X$ or $TQ$ in terms of $Y$, where $DQ$ is drawn parallel to $OT$.

To prove the relation between compensated and ordinary elasticities: Define the elasticity of demand for imports in A,

$$\eta = \frac{RQ}{QM} \frac{TQ}{QM},$$

and the marginal propensity to import in A, $m_a = \frac{RC}{TQ}$.

Then

$$\eta_a - m_a = \frac{RQ}{TQ} - \frac{RC}{TQ} = \frac{CQ}{TQ}.$$

But

$$\frac{CQ}{TQ} = \frac{CQ}{QM} \frac{TQ}{QM},$$

which is the elasticity of demand for imports with the income effect removed, i.e., the compensated elasticity of demand for imports, $\eta'_a$.

are constant. If output in country A increases by $dX^*$ then the change in demand for imports in A is

$$P \frac{dI_A}{dD_A} \frac{dD_a}{dX^*} dX^* = m_a dX^*$$

since expenditure (equals money income) increases by the full amount of the increase in output. Since at constant terms of trade no change occurs in country B, this is also equal to the excess demand for imports in A. This excess demand must be eliminated by a change in the terms
of trade. The criterion is therefore:

\[
\frac{dP}{dX^*} = \frac{m_a}{I(\eta_a + \eta_b - 1)}.
\]

This result is obvious: an increase in the world output of \(X\) must lower the relative world price of \(X\). The only exception (apart from constant costs and incomplete specialization, or perfect substitution in consumption) is the case where the country which has grown spends all of its increased income on its own good \((m_a = 1 - c_a = 0)\).

Equation (22) can be translated into proportional changes by multiplying both sides by \(X\) (output) and dividing by \(P\), with the following result:

\[
\frac{dP/P}{dX*/X} = \frac{m_a X/PI_a}{\eta_a + \eta_b - 1} = \frac{\sigma_a}{\eta_a + \eta_b - 1},
\]

where \(\sigma_a\) is the marginal propensity to spend on imports divided by the average propensity to spend on imports—i.e., the income elasticity of demand for imports. Alternatively we can express the criterion in terms of annual rates of change as follows:

\[
P = \frac{\sigma_a}{\eta_a + \eta_b - 1}
\]

where \(\dot{P}\) is the percentage deterioration of A’s terms of trade per year and \(\dot{R}\) is the annual rate of growth.

By following a similar procedure for country B we can find the annual percentage change in the terms of trade when both countries are growing:

\[
P = \frac{\sigma_a R_a - \sigma_b R_b}{\eta_a + \eta_b - 1}
\]

where \(\sigma_b\) and \(\dot{R}_b\) are, respectively, the income elasticity of demand for imports in B and the annual rate of growth of B.

**Real Income.** A more interesting question is whether or not real income will increase or decrease as a result of a change in productivity. An increase in productivity affects real income in two ways which work in opposite directions. On the one hand real income increases by the full amount of the change in output at constant terms of trade; but, against this may be set the negative income effect of the actual deterioration in the terms of trade. We may very simply derive a measure of the change

\[14\text{ To derive this criterion directly, differentiate } T= I_b(D_b, 1/P) - PI_a(D_a, P) \text{ with respect to } X^* \text{ (output in A). With lending } (T) \text{ constant and zero, this yields}
\]

\[
\left( - \frac{\partial I_b}{\partial(1/P)} - \frac{\partial I_a}{\partial P} - 1 \right) \frac{dP}{dX^*} = m_b \frac{dD_b}{dX^*} - m_a \frac{dD_a}{dX^*}.
\]

Expenditure in B is constant (in terms of B’s good) so \(dD_b/dX^* = 0\); but expenditure in A increases pari passu with output, i.e., \(dD_a/dX^* = 1\). Then, forming elasticities from the terms of the left, we get equation (22) above.
in real income by adding the effects of the change in real income at constant terms of trade, and the reduction in income due to the change in the terms of trade. The change in real income is $dU^*_a = dX_a - IdP$. Dividing by $dX_a^*$ and applying equation (22) we obtain the following criterion:

$$\frac{dU_a}{dX_a^*} = 1 - I \frac{dP}{dX_a^*} = 1 - \frac{ma}{\eta_a + \eta_b - 1}. \quad (26)$$

Real income will be increased by growth provided that $\eta_a + \eta_b > 1 + ma$, i.e., if the sum of the elasticities of demand for imports is greater than unity plus the marginal propensity to import. Note that violation of this condition is consistent with stability. (See Figure 4.)

It will clarify the meaning of (26) if we make use of the relationship used in the previous section between "ordinary" and "compensated" elasticities of demand for imports—in particular, $\eta_a = \eta'_a + ma$. Manipulating (26) we get

$$\frac{dU_a}{dX_a} = \frac{\eta_a - ma + \eta_b - 1}{\eta_a + \eta_b - 1} = \frac{\eta'_a + \eta_b - 1}{\eta_a + \eta_b - 1}. \quad (27)$$

Thus real income will increase in the growing country if $\eta'_a > 1 - \eta_b$, i.e., if the compensated elasticity of demand for imports in the growing country is greater than unity minus the ordinary elasticity of demand for imports in the other country. This criterion may be simplified somewhat by substituting in it the well-known relation between the elasticity of demand for imports and the elasticity of supply of exports ($e$): $\eta_b - 1 = e_b$. The criterion is therefore $\eta'_a + e_b \geq 0.15$

15 This relationship can be derived from the income-expenditure condition. From equation (2), with no lending, we have:

$$x_b/P + \gamma_b = x_b/P + Y_b$$
or

$$Y_b - \gamma_b = \frac{1}{P} (x_b - x_b)$$
or

$$E_b = \frac{1}{P} I_b$$

where $E_b$ represents B's offers of exports. Differentiating, and taking $P$ as initially unity, we have:

$$\frac{P}{E_b} \frac{dE_b}{dP} = \frac{P}{I_b} \frac{dI_b}{dP} - 1,$$

after dividing by $E_b = I_b/P$.

Thus $\epsilon_b = \eta_b - 1$, recalling that

$$\frac{P}{I_b} \frac{dI_b}{dP} = -\frac{1/P}{I_b} \frac{dI_b}{d(1/P)} = \eta_b.$$

Note that when the elasticity of demand is unity the elasticity of supply of exports is zero; this means that the same amount of exports is spent on imports regardless of the terms of trade.

A geometric proof of this relation can easily be got from Figure 3 or from Marshall's analysis [11, pp. 337-38].
FIGURE 4. PRODUCTIVITY, THE TERMS OF TRADE AND REAL INCOME

Quadrant I illustrates trade equilibrium with the initial offer curves, OA and OB, intersecting at Q'. Quadrant II illustrates production and consumption equilibrium in country A: Output in A is initially OZ of X; consumption equilibrium is at C (the length and slope of ZC is equal to the length and slope of OQ').

Suppose that output and expenditure in A increase to OZ' in terms of X. Then at constant terms of trade inhabitants of A demand more of both goods (in the absence of inferior goods), wishing to consume at a point like D. This means that, at any given terms of trade, A demands more imports than before the productivity change so that A's offer curve unambiguously shifts to the right (quadrant I). At the original terms of trade there is an excess demand for Y and a deficit in A's balance of payments equal to RW. The disequilibrium induces a flow of gold from A to B and a deterioration in A's terms of trade.

To connect the ratio of the change in the terms of trade TQ/QM to the increase in output ZZ', i.e., to derive a criterion for the change in the terms of trade, first define the marginal propensity to import in A. Since expenditure increases by VS at constant terms of trade and the demand for imports increases by the vertical distance between C and D, i.e., by RW, we have $m_A = \frac{RW}{VS}$. Now the deficit in A's balance of payments at constant terms of trade is:

$$\frac{dB}{P} = \frac{RW}{VS} \cdot \frac{VS}{ZC} = \frac{m_A}{P} \cdot \frac{dX^*}{P},$$

where $dX^* = ZZ'$. On the other hand we know from the analysis of Figure 1 that:

$$\frac{dB}{P} = QM \cdot \frac{TQ}{QM} \cdot (\eta_a + \eta_b - 1).$$

It follows that:

$$\frac{dP}{dX^*} = \frac{m_A}{Q(\eta_a + \eta_b - 1)}.$$

It is easily seen that the shift in A's offer curve is greater the higher is the marginal propensity to import in A, and that if the latter is zero no shift occurs (all the extra expenditure in A is spent on home goods).

The change in real income in A, as a result of the productivity increase, is HG, measured at pretransfer prices. The criterion the.

$$\frac{dU_a}{dX^*} = GHH \cdot \frac{TQ}{ZC} = 1 - \frac{GJ}{ZC} = 1 - \frac{dP}{dX^*} = \frac{\eta_a + \eta_b - 1}{\eta_a + \eta_b - 1}$$

is zero for small changes in output when ZG'-extended passes through C. The numerator of this criterion can be derived directly by examining the conditions under which a change in the terms of trade to Z'C can be an equilibrium. Note that this result is not possible if OB is elastic.
An alternative, direct method of getting this criterion is to apply the "method of comparative statics." To find whether real income in the growing country increases or decreases first determine the excess demand for imports on the tentative supposition that real income in A is constant. If real income is constant the terms of trade move against A so there is a pure substitution effect, $-\eta \Delta P$, which measures the increase in demand for imports in A. On the other hand the increase in supply of imports forthcoming from B due to the change in the terms of trade is $\epsilon I dP$. The excess demand at constant real income is therefore $-(\eta + \epsilon) I dP$. The deterioration in A's terms of trade which leaves A's real income unchanged is therefore too great or too little to relieve the excess demand due to the productivity change depending on whether $\eta + \epsilon > 0$. The criterion for the change in real income is thus established since, depending on whether the terms-of-trade change has been too great or too little, real income increases or decreases.

It appears then that a country may be worse off with, than without, the improvement in productivity. Growth may be "damnifying." Too rapid growth of the export industries of one country, and the resultant attempt to push exports onto world markets, results in such a large decline in the terms of trade that the negative income effect of the change in relative prices is greater than the positive income effect of the increase in domestic output at constant prices. The case of the group of primary-producing countries readily presents itself. And, if the model were applied to different sectors within a single economy, it might be found that U.S. agriculture is another example.

Nevertheless, this possibility does not present a valid argument against growth. In the first place the conditions under which increasing productivity can affect real income perversely are quite strict: the foreign elasticity of demand must be less than unity and, perhaps, appreciably so if home substitution effects are high. Second (and more important), since world income as a whole increases, by compensation both countries (or sectors) could be made better off than before. Finally, the damnified country can always impose taxes on trade sufficient to reap at least some of the benefits of the productivity increase. Obviously the perverse effect is not possible if the growing country is following an optimum tariff policy since that implies an elasticity of demand in the foreign country greater than unity.

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16 Mill was aware [17, pp. 150-53] that an increase in productivity would lower the commodity terms of trade and even the factorial terms of trade if foreign demand, in the latter case, were inelastic. Edgeworth interpreted [3, p. 10] Mill's passage as indicating that a country could be "damnified" by growth, supplying the necessary assumption to make Mill's analysis correct. The first derivations of criteria (23) and (26) are due to Meade [14, e.g., p. 153] and to Johnson [6] [8].
VI. Taxes and Subsidies on Trade

A tax or subsidy on trade introduces a divergence between foreign and domestic price ratios. Equal taxes on exports or imports create the same divergence between foreign and domestic prices ratios (if trade is balanced) so that the real effects of import and export taxes are symmetrical. A tax on imports at constant terms of trade raises the relative price of imports in the taxing country and therefore draws resources away from export industries into import-competing industries. A tax on exports at constant terms of trade lowers the relative price of exports in the taxing country and thus pushes resources into import-competing industries. With balanced trade the revenues collected by the two taxes are the same. We may therefore speak of trade restriction or trade promotion without specifying whether the tax or subsidy is on exports or imports.17

There are two analytic methods of disposing of the tariff proceeds. We may assume that the government spends the tariff proceeds on the two goods in a given proportion; or we may suppose that tax proceeds are redistributed as income subsidies to consumers. The latter method, which is used here, is simpler because it avoids the necessity of introducing a government demand equation, and does not give rise to asymmetries when dealing with trade subsidies. In the following analysis it should be remembered that we are in fact examining the effects of tariffs combined with this method of disposing of the proceeds.

The Terms of Trade. To determine the effect of a tariff on the terms of trade first compute the excess demand for imports at constant terms of trade. At constant terms of trade the relative price of imports in the tariff-imposing country (A) rises by the full amount of the tariff. Then, with \( t_a \) representing unity plus the ad valorem rate of tariff, the change in demand for imports before redistribution of the proceeds is:

\[
\frac{\partial I_a}{(P_{ta})} d(P_{ta}) dt_a = \frac{\partial I_a}{\delta(P_{ta})} \frac{P_{ta}}{I_a} I_a dt_a = -\eta_a I_a dt_a
\]

assuming initial free trade \( (t_a = 1) \). To this change in demand must be added the increase in demand for imports occasioned by the redistribution of the tariff proceeds, i.e., \( m_a I_a dt_a \). Adding the terms we get the

17 Marshall writes [12, pp. 180–81]: “... The considerations which can be urged for and against the levying of an import tax on a particular commodity differ widely from those appropriate to a particular export tax: and this is perhaps the origin of an opinion, which seems to pervade a good deal of economic discussion, that a general tax on all imports would have widely different effects from a general tax on all exports. In fact the two taxes would have the same effect: provided they were evenly distributed, equal in aggregate amount, and their proceeds were expended in the same way.” He then shows how this can be proved. Bastable, Edgeworth and others were also aware of the symmetry. For a modern treatment see Lerner [10].
excess demand for imports due to the tariff at constant terms of trade, i.e., \((-\eta_a + m_a)Idt_a = -\eta'_a Idt_a\). This excess demand must be eliminated, at the new equilibrium, by a change in the terms of trade. We then have the following criterion:

\[
\frac{dP}{dt_a} = \frac{-\eta'_{a}}{\eta_{a} + \eta_{b} - 1}.
\]

Tariffs normally improve the terms of trade.\(^{18}\)

The degree to which the terms of trade change following a tariff depends on the elasticities. The more elastic is the foreign offer curve the smaller will be the improvement in the terms of trade due to a tariff, and in the limiting case where the foreign offer curve is perfectly elastic, the terms of trade remain unchanged (the only exception). On the other hand if the foreign offer curve is elastic,\(^{19}\) the greater is the compensated elasticity of demand for imports at home the more effective will a given tariff be in improving the terms of trade; in the limiting case where \(\eta'_a\) is infinite the terms of trade will improve in the same proportion as the (ad valorem) tariff. Now both the above propositions are related to the classical notion about the division of the gains from trade between large and small countries. Because small countries tend to be more completely specialized than large countries, it was generally believed that the offer curve of a small country was less elastic than that of a large country. This means that the gain per unit of trade going to a small country was likely to be larger than that going to a large country. Then since the small country already reaped a large proportion of the gain from trade, opportunities for increasing that gain further through tariffs were small. On the other hand large countries, gaining little from trade, could exact a larger gain by imposing tariffs and forcing small countries to trade at a less favorable price ratio.

**The Domestic Price Ratio.** A common motive for trade restriction is the protection of import-competing industries. In order that these industries be protected the domestic relative price of imports (inclusive of

\(^{18}\) To derive (28) directly differentiate:

\[
T = 0 = I_b(D_b, 1/P) - P I_a(D_a, P_t_a)
\]

with respect to \(t_a\). With \(D_b\) constant this yields:

\[
(\eta_{a} + \eta_{b} - 1)I \frac{dP}{dt_a} = P \frac{\partial I_a}{\partial D_a} \frac{dD_a}{dt_a} - \frac{\partial I_a}{\partial (P_t_a)} = (m_a - \eta_a)I
\]

since \(dD_a/dt_a=I\); i.e., expenditure in A rises by the value of the tariff proceeds.

The qualitative direction of change in the terms of trade was admitted by Ricardo and known to most of the later classical economists. The algebraic criterion can be got from Meade's analysis [14].

\(^{19}\) Note that if the foreign offer curve is inelastic the terms of trade may improve by more than the tariff, provided that the home offer curve is not perfectly elastic; if the latter is perfectly elastic the maximum change in the terms of trade is equal to the rate of the tariff.
the tariff) must rise. We shall therefore develop a criterion for the change in the domestic price ratio following a tariff.

The domestic price ratio in country A is \( P_t a \) (where \( t_a \) is unity plus the ad valorem rate of tariff). We are interested in how \( P_t a \) will change as a result of an increase in \( t_a \), i.e., in the sign of

\[
\frac{d(P_t a)}{d t_a} = P + t_a \frac{d P}{d t_a},
\]

where \( P \) and \( t_a \) are initially equal to unity. Substituting for \( dP/dt_a \), from equation (26) we obtain the following criterion:

\[
\frac{d(P_t a)}{d t_a} = 1 - \frac{n_a'}{n_a + n_b - 1} = \frac{n_b + m_a - 1}{n_a + n_b - 1}.
\]

A tariff raises the domestic price of imports if the sum of the foreign elasticity of demand for imports and the domestic marginal propensity to import is greater than unity. The only exception is the case previously mentioned where the home offer curve is infinitely elastic, in which case the domestic price ratio remains unchanged. (See Figure 5.)

The meaning of the criterion will be clarified if we consider the excess demand for imports on the assumption that the domestic price ratio remains unchanged. In that event the terms of trade improve by the full amount of the tariff, so that the change in supply of imports from B is

\[-b \frac{d P}{d t_a} = (1 - \eta_b) dP/dt_a.
\]

On the other hand the redistribution of the tariff proceeds and the resulting increase in domestic expenditure in A increases the demand for imports by \( m \frac{d P}{d t_a} \). Subtracting the change in supply of imports from B from the increase in demand for imports in A we get the excess demand for imports at constant domestic prices, i.e.,

\[\eta_b + m_a - 1 \frac{d P}{d t_a}.
\]

Now if the foreign demand is less than unit-elastic (foreign supply elasticity is negative) an improvement in A's terms of trade results in an increase in supply of imports from B; but against this must be set the increased demand for imports in A resulting from the spending of the tariff proceeds. If the former effect is greater than the latter, the relative price of imports must fall. If, for example, the foreign elasticity of supply were \(-0.6\) (implying an elasticity of demand equal to \( -0.4 \)) while the domestic marginal propensity to import were \( 0.5 \), the tariff would cause, at constant domestic prices, an excess supply of B's good equal to \( 0.1 \frac{d P}{d t_a} \); to eliminate this excess supply of B's good, the relative (tariff-inclusive) price of imports must fall, and the terms of trade must improve by more than the tariff.

20 The classical economists, many of whom tried to determine whether a country gained more or less than the amount of the tax, generally employed the criterion \( \eta_b > 1 \), assuming, implicitly or explicitly, that the tax proceeds were spent on domestic goods, or that the tax was on the transit of goods that would be re-exported. For modern discussions see Lerner [10] and Metzler [16].
FIGURE 5. Tariffs, the Terms of Trade and the Domestic Price Ratio

The offer curves of A (not drawn) and B (OB) intersect at the free-trade equilibrium \( Q \). Suppose that this equilibrium is disturbed by the imposition of a tariff by A's government. At constant terms of trade the price of \( Y \) in A rises by the full amount of the tariff \( LO/OM \) and trade equilibrium in A moves to a point on A's original offer curve, such as \( H \). But when tariff revenues equal (approximately) to \( OL \) in terms of \( X \) are redistributed to consumers the demand for both goods increases. The equilibrium point for A at constant terms of trade therefore moves to a point on A's revenue-redistributed offer curve, such as \( J \). At constant terms of trade the tariff-cum-income subsidy results in an excess supply of \( Y \) and a surplus in A's balance of payments equal to \( QW \) since the new equilibrium at \( J \) must be below \( Q \) if substitution effects are positive. Gold therefore flows into A from B and A's terms of trade improve until the excess supply \( WQ \) is relieved. Assume that the new equilibrium is at \( Q' \).

The excess demand for imports in A at constant terms of trade is:

\[
-dB = WQ = \frac{WQ}{QM} QM = \frac{WQ}{QM} \frac{OM}{OL} \frac{OL}{QM} QM = \eta_A' dl_a.
\]

On the other hand we have:

\[
\frac{dT}{P} = WQ = WR - TR - TQ = \left[ \frac{WR}{QM} \frac{QM}{TQ} - \frac{TR}{QM} \frac{QM}{TQ} - 1 \right] \frac{QM}{TQ}
\]

\[
\left( \eta_a + \eta_b + 1 \right) \frac{dP}{P}.
\]

It follows that:

\[
\frac{dP}{dl_a} = -\eta_a' \frac{\eta_a}{\eta_a + \eta_b - 1}.
\]

Note that \( \eta_a \) refers to the elasticity of the revenue-redistributed offer curve in A.

For the domestic price ratio in A to be unchanged at the new equilibrium, as a result of the tariff, the slope of the A-indifference curve at \( Q' \) must be the same as the slope of the indifference curves of A and B at \( Q \). But in that case the marginal propensity to import in A is equal to \( RQ/TQ \) (for small tariffs); and since

\[
e_5 = \eta_b - 1 = -\frac{RQ}{QM} \frac{QM}{TQ} = \frac{RQ}{TQ},
\]

it follows that \( \eta_a - m_a - 1 = 0 \). This is the borderline case. It is easily seen that the slope of A's indifference curve at \( Q' \) is flatter or steeper than that at \( Q \) as \( m_a \geq \eta_b - 1 \).
This possibility is consistent with stability. It means that a tariff may have an adverse protective effect. To protect the domestic industry imports must be subsidized instead of taxed! Under no circumstances, however, would a country ever find it beneficial in fact to subsidize imports to protect the domestic industry. For the adverse protective effect to occur the foreign demand must be inelastic, and in that case a tariff must always result in an improvement in national welfare (more imports are obtained for fewer exports). If an optimum tariff policy is being followed a further increase in the tariff always raises the relative price of imports in the tariff-imposing country and thus has a normal protective effect.

Tariff Changes in Both Countries. If both countries adjust their tariff rates the extent and direction of the disequilibrium depends on the size of the tariff changes and the elasticities of demand. In bilateral tariff negotiations it may be useful to know what adjustment in the tariffs of both countries will leave the balance of payments unaltered. If, for practical reasons, changes in the terms of trade (through exchange rate or wage changes) must be ruled out, countries embarking on, say, a customs union experiment may need to know the rate at which each can reduce tariffs so as to leave the balance of payments unchanged. The answer to this question can readily be obtained from equation (28) making appropriate adjustments for country B. If we write $P$ as the annual deterioration of country A's terms of trade, $\eta_a$ as the annual change in A's tariff and $\eta_b$ as the annual change in B's tariff we can obtain the following criterion:

$$P = \frac{\eta_a \Delta t_b - \eta_b \Delta t_a}{\eta_a + \eta_b - 1}.$$  

In order to prevent any change in the terms of trade the numerator must be zero; tariffs must then be changed at a rate inversely proportional to the compensated elasticities of demand for imports. [Note that equation (28) applies with a changed sign to subsidies since subsidies are simply negative tariffs.]

Income Transfers and Trade Taxes. Suppose the authorities in one country wish to make a grant to another country but, for political or other reasons, are unable to make the gift official. Alternatively, suppose that one country wishes to exact a transfer of real income from another country without imposing a formal tribute. Can this be accomplished efficiently by changes in trade taxes? (See Figure 6.)

To show that it can consider first the relation between trade taxes and subsidies. Suppose that country A subsidizes exports (or imports) while country B taxes imports (or exports). In that case the same goods are being taxed, the only difference being that the customs duties are
Initial equilibrium is at $Q$ on the contract curve $KK$. Suppose that A subsidizes, and B taxes, trade at the same rate $OT/TM$. Then A's and B's offer curves bend down to $OA'$ and $OB'$, respectively, intersecting at $Q'$ (they still originate from $O$). Since price ratios remain the same in both countries $Q'$ must be on the contract curve. The slope of the indifference curves at $Q'$ are, from the transfer analysis, greater or less than the slope at $Q$ depending on whether the sum of the marginal propensities to import is greater or less than unity. The terms of trade are now $OQ'$, necessarily worse for A if the system is stable. Real income in A falls to the same extent as if A had made a gift of $OT$ of $X$ to B.

Suppose now that only B has a tax (equal to $OT/TM$) so that trade equilibrium is at $S$. To restore efficiency (Pareto optimum), A can impose a subsidy equal to B's tariff, attaining the equilibrium (worse for A, better for B) $Q'$. A more interesting possibility is for A to bribe B to remove the tariff, the value of the bribe being the transfer necessary to effect an equilibrium between $W$ and $L$ (better for both than $S$). Or if A is already receiving gifts from B then tariff reduction in B and the elimination of gifts to A (i.e., "Trade, not Aid") can make both A and B better off.

Collected by officials of different nationality. Trade is subsidized in A and taxed in B. Now if the increase in trade subsidies in A is equal to the increase in trade taxes in B this combined policy is equal to a transfer of income from A to B equal to the value of the tax-subsidy payments. Since the tax in B cancels the subsidy in A, price ratios in the two countries must be the same. The change in real income, evaluated at the original price ratio, in each country is given by:

$$-dU_a = dU_b = IdP = \frac{(-\eta_a \ell_a + \eta_b \ell_b)I}{\eta_a + \eta_b - 1}.$$
Now if \(-dt_a = dt_b = dt\) we have:

\[
\frac{-dU_a}{Idt} = \frac{-dU_b}{Idt} = \frac{\eta_a + \eta_b'}{\eta_a + \eta_b - 1}.
\]

But (31) is the same as the criterion for the change in real income after transfer [equation (19) above] if \(Idt\), the value of the tax-subsidy receipts, is substituted for the transfer.

**VII. Consumption and Production Taxes**

Taxes on commodities, as distinct from taxes on trade, make it necessary to distinguish between consumers’ and producers’ price ratios. A consumption tax or subsidy creates a divergence between the price ratio facing consumers in the taxing country and all other price ratios, while a production tax causes a discrepancy between the price ratio facing producers and all other price ratios.

There are eight taxes and subsidies in each country which are possible, but it will not be necessary to consider more than two. A subsidy is a negative tax so that we need only consider taxes. And a tax on one good is equivalent to a subsidy on the other good because of our assumptions about the disposal of tax proceeds and the financing of subsidy payments. Because of these assumptions an equal tax on the two goods has no effect on equilibrium; this follows because each tax is combined with an income subsidy, and an income subsidy has the same effect as an equal subsidy (or an equal reduction in taxes) on the two goods. But if an equal tax on the two goods does not affect equilibrium, then neither does a tax on one of the goods combined with the elimination of a subsidy of equal amount on the other good—hence it follows that a tax on one of the goods is equivalent to a subsidy on the other. Thus we need only consider one consumption tax and one production tax.

**Consumption Taxes and the Terms of Trade.** To find the effects of a consumption tax on the terms of trade first determine the excess demand caused by the tax at constant terms of trade. Let us suppose that a tax is imposed by country A on the imported good, \(Y\). Then at constant terms of trade the price of \(Y\) to consumers in A rises by the amount of the tax. Before the redistribution of the tax proceeds the change in demand for \(y\) in A, at constant terms of trade, is

\[
\frac{\partial y_a}{\partial (Pt_{eya})} = \frac{Pt_{eya}}{y_a} y_a dt_{eya} = -\eta_y a y_a dt_{eya}
\]

where \(t_{eya}\) is unity plus the rate at which commodity \(Y\) is taxed in A.
and \( \eta_{y_a} \) is the elasticity of demand for \( y \) in A.\(^{21}\) (\( P \) and \( \xi_{y_a} \) are both initially taken to be unity.) Now the tax proceeds amount to \( y_a dl_{y_a} \) so that the increase in demand for \( y \) due to their disposition is \( m_a y_a dl_{y_a} \). Adding the two effects we get the excess demand at constant terms of trade,

\[
(m_a - \eta_{y_a})y_a dl_{y_a} = -\eta_{y_a} y_a dl_{y_a},
\]

where \( \eta_{y_a}' \) is the compensated elasticity of demand for \( y \) in A, i.e., the elasticity of demand for \( Y \) after consumers have been compensated for the change in real income implicit within the price change. We now have the following criterion:

\[
(32) \quad \frac{dP}{dl_{y_a}} = -\frac{y_a}{I} \frac{\eta_{y_a}'}{\eta_a + \eta_b - 1}.
\]

The compensated elasticity term is positive (it represents the elasticity of a consumption indifference curve) so that a consumption tax on the imported good generally improves the terms of trade of the taxing country. This conclusion is to be expected since a tax on the imported good diverts demand away from that good, thereby causing an excess world supply.

There are some special cases we may consider: (1) in the unusual case where \( \eta_{y_a}' \) is zero—implying no substitution in consumption (a kinked consumption indifference curve at that point)—the terms of trade do not change; (2) if the foreign offer curve is perfectly elastic any excess demand can be eliminated by shifts in production or consumption at constant cost in the foreign country so there results no change in the terms of trade; (3) if the domestic offer curve is perfectly elastic (\( \eta_a \) is infinite) it is now necessary to know whether this is due to perfect substitution in production (incomplete specialization at constant cost) or perfect substitution in consumption; if the former is the case the denominator of (32) is infinite, so the terms of trade do not change; but if

\(^{21}\) The elasticity of demand for the imported good is never larger than the elasticity of demand for imports. The exact relation can be derived from the definition of the demand for imports. From \( I_a = y_a - Y_a \) we get, by differentiation,

\[
\frac{\partial I_a}{\partial P} = \frac{\partial y_a}{\partial P} - \frac{\partial Y_a}{\partial P}
\]

and

\[
-\frac{P}{I_a} \frac{\partial I_a}{\partial P} = -\frac{\partial y_a}{\partial P} \frac{y_a}{I_a} + \frac{\partial Y_a}{\partial P} \frac{Y_a}{I_a}
\]

whence

\[
\eta_a = \eta_{y_a} \frac{y_a}{I} + \epsilon_{y_a} \frac{Y_a}{I}
\]

where \( \epsilon_{y_a} \) is the elasticity of supply of \( Y \) in A. (A similar result, holds for country B.) The elasticities \( \eta_{y_a} \) and \( \eta_a \) coincide only when there is no home production of the imported good.
the goods are perfect substitutes in consumption both the denominator and the numerator are infinite so that the terms of trade change by the full amount of the tax; (4) if there is no domestic production of the imported good the tax has the same effect as a tariff \( (\gamma_a = 1 \text{ and } \eta_{ya}' = \eta_y') \).

This analysis applies also to a subsidy on the consumption of the good which is exported and, with a change of sign, to a subsidy on the imported good or tax on the exported good.

**Consumption Taxes and the Ratio of Market Prices.** Does a consumption tax necessarily raise the market price of the taxed good relative to that of the untaxed good? By analogy to the effect of a tariff on the domestic price ratio we should not expect this to be so. The market (relative) price of importables is \( \frac{P_{cy}}{I_{cy}} \), which we assume to be initially unity. The change in this price ratio due to the tariff is:

\[
\frac{d(P_{cy})}{dI_{cy}} = 1 - \frac{dP}{dI_{cy}} = 1 - \frac{\eta_a}{I_{cy}} \eta_{ya} \eta_y - 1
\]

(33)

\[
\eta_y + m_a + \frac{\eta_{ya}'}{I_{cy}} - 1
\]

where \( \eta_{ya}' \) is the compensated elasticity of supply of \( Y \) in \( A \), and represents the elasticity of the production transformation curves.22 Only by

---

22 The compensated elasticity of supply requires some explanation. From the two relations:

\[
\begin{align*}
\eta_a &= \eta_a' + m_a, \\
\eta_y &= \frac{\eta_y}{I} + \frac{\epsilon_{ya} Y_a}{I}
\end{align*}
\]

we can obtain:

\[
\eta_a = \eta_a - m_a = (\eta_y - \eta_y) \frac{\eta_y}{I} + (\epsilon_{ya} + m_a) \frac{Y_a}{I}.
\]

It can now be shown that \( \eta_y - m_a = \eta_y' \), the compensated elasticity of demand for \( Y \) in \( A \); and that \( \epsilon_{ya} + m_a = \epsilon_{ya}' \), the compensated elasticity of supply of \( Y \) in \( A \). We have \( dI_a = dY_a - dY_a \) from the definition of the demand for imports. Now \( dI_a \) contains an income effect equal to \( -m_a Y_a dP \) where \( -I_a dP \) measures the change in real income of country \( A \) looked at as a whole. The term \( dY_a \) contains an income effect equal to \( -m_a Y_a dP \), where \( -Y_a dP \) measures the change in real income of people of country \( A \) looked at in their role as consumers alone. Finally the term \( dY_a \) contains an income effect equal to \( m_a Y_a dP \) where \( Y_a dP \) measures the change in real income of producers of \( Y \) in country \( A \). We now have the following relations:

\[
\begin{align*}
dI_a &= (dI_a)' - m_a Y_a dP; \quad dY_a = (dY_a)' - m_a Y_a dP; \quad dY_a = (dY_a)' + m_a Y_a dP
\end{align*}
\]

where the primes denote pure substitution effects. Substituting in \( dI_a = dY_a - dY_a \) we get

\[
(dI_a)' - m_a Y_a dP = (dY_a)' - m_a Y_a dP - [(dY_a)' + m_a Y_a dP].
\]

The income effects on the two sides cancel and the proof of the relation between compensated elasticities follows readily. Dividing by \( I_a dP \), multiplying by \( P \), and changing signs we obtain

\[
\eta_a' = \frac{\eta_y}{I} + \frac{\epsilon_{ya} Y_a}{I}
\]

where the primes denote that the elasticities contain no income effects.
this term does the last criterion in (33) differ from the criterion for a change in the domestic price ratio after the imposition of a tariff [see equation (29)]. If a tariff will raise the domestic relative price of imports so will a consumption tax on the import good. The converse is not true. Even if the foreign offer curve is inelastic and the domestic marginal propensity to import is low, high substitution effects in production will be sufficient to ensure a rise in the tax-inclusive price of importables.

Production Taxes and the Terms of Trade. To find the effects of a production tax on the terms of trade first consider the excess demand caused by the production tax at constant terms of trade. If a tax on the production of the import good is imposed and the proceeds of the tax are redistributed to producers there remains only a pure substitution effect, a movement along the production-possibility curve. At constant terms of trade (which is also the price ratio facing domestic consumers) the excess demand for imports, after the redistribution of the proceeds, is equal to:

\[ \epsilon_{ya} Y_a dl_{pva} \]

where \( l_{pva} \) is equal to unity plus the tax, and \( \epsilon_{ya} \) is the elasticity of the transformation curve. This excess demand must be eliminated by a worsening of A's terms of trade. The criterion is therefore:

\[ \frac{dP}{dl_{pva}} = \frac{Y_a}{I} \frac{\epsilon_{ya}}{\eta_a + \eta_b - 1} \]

Since with increasing opportunity costs \( \epsilon_{ya} \) is always positive the terms of trade of the taxing country fall. Again this applies to a production subsidy on the exported good and, with a change of sign, to a production tax on the exported good. This conforms to common sense. A tax on the production of any good decreases the production of that good and increases the production of the other good causing a rise in the relative world price of the taxed good.

Production Taxes and Relative Prices at Factor Cost. The price ratio

These elasticities have a simple interpretation: \( \eta_a' \) is the elasticity of a trade-indifference curve; \( \eta_{pa}' \) is the elasticity of a consumption-indifference curve; and \( \epsilon_{pa} \) is the elasticity of a production-indifference (production-possibility) curve. The identification is formally valid for small changes only.

Criterion (32) can be derived directly by differentiating:

\[ T = 0 = I_s(D_b, 1/P) - P[y_a(D_a, P_{t_{pva}}) - Y_a(P)] \]

noting that \( D_b \) is constant and that

\[ \frac{dD_a}{dt_{pva}} = \eta_a. \]
facing producers is $P/l_{pyy}$, which is initially taken to be unity. This will change as a result of a tax depending on the sign of:

$$\frac{d(P/l_{pyy})}{dl_{pyy}} = \frac{dP}{dl_{pyy}} - 1 = \frac{Y_a}{I} \frac{\eta_a + \eta_b - 1}{\eta_a + \eta_b - 1} = \frac{-\left(\eta_b + m_a + \frac{\eta_b}{I} - 1\right)}{\eta_a + \eta_b - 1}.$$  

(35)

The analogy to equations (29) and (33) above readily presents itself.23

Relation between Commodity Taxes and Trade Taxes. A tariff, at constant terms of trade, raises the price of imports to both consumers and producers in the taxing country by the full amount of the tariff. Any other system of taxes which does the same thing will have the same effect on the terms of trade as a tariff. Thus taxes on trade can be duplicated by taxes on commodities. We can establish these relations either by showing that the tax combination affects the price ratios facing consumers and producers in the same way as a trade tax, or by adding the effects of the criteria obtained above in each case. By either method it is readily shown that a tariff (on Y) or an export tax (on X) is equivalent in real terms to: (a) a consumption tax on Y plus a production subsidy on Y; (b) a consumption tax on Y plus a production tax on X; (c) a consumption subsidy on X plus a production subsidy on Y; (d) a consumption subsidy on X plus a production tax on X. And because trade subsidies are negative trade taxes, an export subsidy (on X) or an import subsidy (on Y) can be duplicated by: (e) a consumption subsidy on Y plus a production tax on Y; (f) a consumption subsidy on Y plus a production subsidy on X; (g) a consumption tax on X plus a production tax on Y; (h) a consumption tax on X plus a production subsidy on X. From these relations it follows that: (1) the effects of devaluation can be duplicated or frustrated by changes in commodity taxes and subsidies (since devaluation is equivalent to an import tariff plus an export subsidy); (2) the optimum tariff can be duplicated by commodity taxes; and (3) income transfers can be duplicated by changes in

23 Criterion (34) can be derived directly by differentiating:

$$T = 0 = I_b(D_b, 1/P) - P\left[\gamma_a(D_a, P) - Y_a\left(\frac{P}{l_{pyy}}\right)\right]$$

noting that

$$\frac{dD_a}{dl_{pyy}} = Y_a.$$

To my knowledge the effects of consumption and production taxes on the terms of trade have not been formally treated in the literature before, although the general direction of their influence is indicated in many classical writings. See, for example, Viner [24, p. 363].
commodity taxes in both countries. A further application is to customs unions; an agreement over tariff reduction has little force if it is not combined with agreement over the domestic tax structures.

Consideration of the commodity-tax structure is necessary before evaluating the desirability of tariff reductions. If there are commodity taxes and subsidies in each country none of the well-known welfare propositions of international trade theory holds. In particular from a "free-trade" (i.e., no trade taxes) position, it can be shown that, if there are commodity taxes and subsidies: (1) both countries simultaneously may be better off without, than with, trade; (2) a country may gain by a deterioration in its terms of trade even if the initiating cause occurs in the foreign country; (3) a small tariff may worsen the welfare of the tariff-imposing country even if the foreign offer curve is not infinitely elastic; and (4) the imposition of a tariff may simultaneously improve the welfare of both countries. These propositions follow because commodity taxes overextend or underextend trade.

VIII. Other Mechanisms of Adjustment

Thus far we have dealt with the effect of policy changes on the terms of trade, the latter adjusting through price-level or exchange-rate variations. It was argued earlier that authorities may adopt other policies which prevent, or render unnecessary, changes in the terms of trade. This would be the case if authorities pegged the exchange rate and stabilized domestic price levels, relying on, say, trade controls to correct disequilibria. The purpose of this section is to show how the results already obtained can be applied to other mechanisms of adjustment. The procedure to be followed is the same as before: First state the postulate on which dynamic behavior is based, deduce the condition of dynamic stability, and then examine the excess demands caused by the policy changes.

Fiscal Policy and Capital Movements. Suppose that authorities peg the exchange rate and stabilize the domestic price level in each country. The price level may be stabilized in a variety of ways but the simplest for our purpose is to suppose that authorities inflate domestic expenditure by means of a budget deficit when there is deflationary pressure (excess supply of its export good) and deflate domestic expenditure by means of a budget surplus when there is inflationary pressure (excess demand for its export good). We may assume that the deficits and surpluses are financed and disposed of by drawing on or accumulating credits with an international agency—say, the International Monetary Fund. Now since an excess supply of one country’s good implies an excess demand for the other country’s good it follows that one country will be borrowing at the time another country is lending; and because of
the identity of income (including loans) and expenditure, the rate of lending by one country is equal to the rate of borrowing in the other country, and both are equal (with appropriate signs) to the rate at which the budgets are out of balance.

Whether or not a system based on these rules is stable depends on the effectiveness of the deflation-inflation policy in relieving excess demand for the deflating country's good and excess supply of the inflating country's good. But it is easily seen that this is equivalent to whether a transfer from one country to another will cause an excess supply of the transferring country's good. The system is therefore stable or unstable depending on whether the sum of the marginal propensities to import is less than or greater than unity. The term \(1 - m_a - m_b\) also gives the denominator of the criterion showing the direction and amount of lending required to eliminate a given excess demand. (See Figure 7.)

To determine the effects of policy changes on lending in a system obeying the above rules we (as before) find the excess demand due to the policy change with no lending. For example, the excess demand for imports due to a tariff in country A is \(-\eta_a I d t_a\). The change in the trade balance and lending of country A is therefore:

\[
\frac{dT}{dt_a} = \frac{\eta_a I}{1 - m_a - m_b};
\]

if the system is stable the tariff improves the trade balance.\(^{24}\) Or we may consider the change in lending and the trade balance due to an increase in productivity in, say, country B:

\[
\frac{dT}{dY^*_B} = \frac{m_b}{1 - m_a - m_b};
\]

assuming stability, country A must lend to country B to maintain equilibrium in the balance of payments.

In a similar fashion we can find the effects on lending of all the policies discussed in previous sections. It may be helpful to consider two cases. Suppose that country A devalues its currency. Applying the same method we find that the criterion for the change in the balance of trade and lending is:

\[
\frac{dT}{dP} = \frac{(\eta_a + \eta_b - 1)I}{1 - m_a - m_b}.
\]

It should be noticed that (38) is the reciprocal of (18), the criterion for the change in the terms of trade after transfer. The interpretation is

---

\(^{24}\) This criterion has been used by Meade [14, p. 155] and derived geometrically by Ozga [21] although in neither case is a distinction made between stable and unstable situations. Ozga's analysis, which was presented to a seminar in London in 1956, greatly improved my own geometrical representations.
Let equilibrium be initially at $Q$ with the government of $A$ lending or giving to the government of $B$ the annual payment $OH$. It is assumed that exchange rates are fixed and that each government, by means of fiscal policy, stabilizes export price levels.

Now suppose that the equilibrium is disturbed by, say, a private flow of capital of $HL$ from $B$ to $A$, and that this induces an excess of saving over investment in $B$, and an excess of investment over saving in $A$, equal to the transfer. Now if the sum of the marginal propensities to import is less than unity, as in the diagram, the Engel curve of $A$ ($AA$) must be flatter than the Engel curve of $B$ ($BB$); the capital flow therefore induces an excess demand for $A$'s good and an excess supply of $B$'s good, and a surplus in $A$'s and a deficit in $B$'s balance of payments.

To correct the disequilibrium, $A$'s government deflates expenditure by means of a budget surplus and turns the proceeds over to the IMF; and $B$'s government inflates expenditure by means of a budget deficit borrowing from the IMF. This process continues until the inflationary pressure in $A$ and the deflationary pressure in $B$ are eliminated, i.e., until the equilibrium $Q$ and the net lending position $OH$ are restored. By similar analysis it can be shown that a movement of capital from $A$ to $B$ in excess of $OH$ (say to $OK$) will cause deflationary pressure in $A$ and inflationary pressure in $B$, necessitating government action in each country to eliminate the disequilibrium. In either case the equilibrium $Q$ is stable.

But if the sum of the marginal propensities to import exceeds unity the system is unstable. This may be seen by considering again a movement of capital from $B$ to $A$ of $HL$. This time the capital movement causes an excess supply of $A$'s good and an excess demand for $B$'s good. $A$'s government therefore inflates expenditure and $B$'s government deflates expenditure, moving the system ever further from equilibrium.

As a practical problem it would be necessary to distinguish between the Engel curves appropriate for different types of transfer.

fundamentally different. In (18) the stability condition is that the sum of the elasticities is greater than unity, while in (38) the stability condition is that the sum of the marginal propensities to import is less than unity. In (18) lending induces—because of the "rules of the gold-standard- (or flexible-exchange-rate-) game"—a change in the terms of trade; in (38) devaluation induces—because of the "rules of the IMF game"—a change in the balance of trade and lending. An interesting result is the following: If the IMF system is unstable the gold standard (or flexible exchange) system is stable; and if the gold standard (or
flexible exchange) system is unstable the IMF system is stable. Instability of one system therefore implies stability of the other system, though not vice versa. This relation holds because the sum of the marginal propensities to import is less than the sum of the elasticities of demand for imports.

Finally, consider a trivial case. A change in capital exports has no ultimate effect on net lending! In the IMF system there is only one equilibrium rate of lending (in the absence of other trade policy changes), just as, in the classical system, there is only one equilibrium value of the terms of trade (assuming that the equilibrium is unique). This trivial case is cited for purposes of comparison with the classical contention that devaluation, from a position of equilibrium, does not change the terms of trade or the balance of trade: instead, it initiates price level changes which restore the equilibrium terms of trade. A displacement of the variable of adjustment from equilibrium initiates dynamic forces which induce a return to equilibrium.

Similar analysis can be applied to systems of adjustment based on tariff, tax or productivity changes.

IX. Summary

The results of the preceding analysis may all be summarized by introducing all policy parameters into the balance of payments equation and differentiating. We obtain:

\[
(1 - m_a - m_b) dT - I(\eta_a + \eta_b - 1) dP - I\eta_a' d\ell_a + I\eta_b' d\ell_b - \gamma a \eta a' d\ell a + \eta b \eta b' d\ell b
\]

\[
+ x_\eta \eta d\ell c + Y_a \epsilon_a' d\ell p_a - X_b \epsilon_b' d\ell p_b + m_a dX_a^* - m_b dX_b^* = 0
\]

where the same terminology is used as before except that effective tax rates are used. (Thus \(d\ell_a\) and \(d\ell_b\) refer to the effective rate at which trade taxes are changed in A and B; changes in trade subsidies are subtracted from changes in trade taxes. Similarly \(d\ell ca\) and \(d\ell cb\) represent the effective rate at which consumption taxes or subsidies are changed in A and B; a tax on the consumption of import goods combined with an equal tax on the consumption of home goods would leave the effective rate unchanged. Similarly for production taxes \(d\ell pa\) and \(d\ell pb\).)

The policy equation (39) shows the relation between policy changes which are necessary to maintain equilibrium in the system; it can be used to show the policy changes which are necessary to offset the disequilibrium caused by other policies. Suppose, for example, country A wishes to know the rate at which it must tax import goods in order to relieve a disequilibrium caused by an increase in productivity in the foreign country. To find the answer set all policy changes except \(d\ell ca\) and \(dX_b^*\) equal to zero. This leaves the equation

\[-\gamma a \eta a' d\ell ca - m_b dX_b^* = 0\]
and the answer
\[
\frac{dt_a}{dX_b} = \frac{m_b}{y_a\eta_{ab}'}.
\]

The productivity change in B causes a surplus in A's balance which can be relieved by a reduction in the rate at which consumption of import goods (export goods) are taxed (subsidized) in A. Any other relation between two or more policy changes can in this way be determined.\textsuperscript{25}

X. Extension of the Model: The Multiple Country Case

The propositions now established have been derived from a model consisting of only two goods and two countries. The traditional use of this model in international trade literature is based on a belief that it suggests "theorems which may be seen to admit of extension to more concrete cases" \cite[p. 31]{Mundell1963}. In this final section general results are derived and it is shown that in at least one important case the propositions previously established for two countries hold for an arbitrary number of countries.

Suppose that there are \(n+1\) countries, and let all prices and balances of payments be expressed in terms of the currency of country \(0\) (e.g., dollars). If prices (of export goods or currencies) are flexible then the balance of each country depends on all prices. The conditions of equilibrium can be written as follows:

\[
B_1(P_1, \ldots, P_n; \alpha) = 0
\]

\[
\cdots \cdots \cdots \cdots \cdots
\]

\[
B_n(P_1, \ldots, P_n; \alpha) = 0
\]

where \(\alpha\) is a parameter representing a particular policy.\textsuperscript{26} Note that only

\textsuperscript{25} All of the criteria contained in equation (39) are capable of simple geometric proofs when the variables are taken two at a time.

The above analysis of policy changes has been expounded on the assumption that transport costs are absent; for a geometrical representation of transport costs employing Marshallian offer curves see Mundell \cite{Mundell1963}.

\textsuperscript{26} Let \(x_{rs}\) and \(X_{rs}\) be, respectively, consumption and production of the \(r\)th good in the \(s\)th country, and let \(T_s\) be the capital exports of country \(s\). Suppose that there are \(n\) countries and \(m+1\) goods. Then a general model can be expressed by the following equations:

\[Y_s - D_s = \sum_{r=1}^{m} P_r(X_{rs} - x_{rs}) = T_s \quad (s = 1, 2, \ldots, n)\]

The national "budget" equations: Income - Spending = Lending, for each country. If these equations are satisfied each country is on its \(m\)-dimensional offer curve.

\[\sum_{s=1}^{n} (X_{rs} - x_{rs}) = 0 \quad (r = 1, 2, \ldots, m)\]

The market clearing equations: World Supply = World Demand, for every good. Notice that the excess supply function of the numeraire, commodity 0 (e.g., gold), is omitted; if equations (1) are satisfied, then the last equation in (2) can be deduced from the others (or vice versa).
$n$ equations are required since we are dealing with *normalized* prices; the balance of payments of country 0 follows from what may be called *Cournot's Law.*\(^{27}\)

We wish to determine the effects of a shift in the policy $\alpha$ on the equilibrium set of prices so we differentiate equations (40) with respect to $\alpha$. With appropriate choice of units of currencies or goods we get:

$$
\begin{align*}
\frac{dP_1}{d\alpha} + \cdots + \frac{dP_n}{d\alpha} &= - \frac{\partial B_1}{\partial \alpha} \\
\frac{dP_1}{d\alpha} + \cdots + \frac{dP_n}{d\alpha} &= - \frac{\partial B_n}{\partial \alpha}
\end{align*}
$$

(41)

where $b_{ij} = \partial B_i/\partial P_j$ is the change in the balance of the $i$th country due to a change in the price of the goods of the $j$th country. Solving for $dP_i/d\alpha$ we obtain,

$$
\frac{dP_i}{d\alpha} = - \frac{\partial B_i}{\partial \alpha} \frac{\Delta_{1i}}{\Delta} + \cdots - \frac{\partial B_n}{\partial \alpha} \frac{\Delta_{ni}}{\Delta} = - \sum_{j=1}^{n} \frac{\partial B_j}{\partial \alpha} \frac{\Delta_{ji}}{\Delta}
$$

(i = 1, \cdots, n)

(42)

Demand for the $r$th good in the $s$th country depends on the level of spending and the prices both expressed in terms of money, commodity 0.

$$
X_{rs} = x_{rs}(D_s, P_1, \cdots, P_m)
$$

(r = 0, 1, \cdots, m) \quad (s = 1, \cdots, n)

(3)

Supply of the $r$th good in the $s$th country depends on the prices.

Domestic expenditure is assumed to be linked to lending and other policy variables. If we add to this system $n$ equations of the form $D_i = D_i(T_s, \alpha)$, where $\alpha$ is any policy, then we have a system of $2(mn+n)+m+n$ equations, and $2(mn+n)+m+2n$ unknowns. By specifying $n$ parameters representing the capital exports of each country, the system becomes determined.

In the text the $P$'s refer to the price level of each country's exports (exchange rates constant), or the price of each country's currency (price levels constant). If each country exports only one product, and the budget equations (1) are satisfied, then the partial derivatives $\partial B_i/\partial P_j = b_{ij}$ [used in equation (42)] can be identified with the world elasticity of demand and supply of the exports of the $i$th country with respect to a change in the price of the exports of the $j$th country, making use of the following relations:

$$
B_i = \text{Value of Exports} - \text{Value of Imports} = (\text{Foreign Demand} - \text{Domestic Excess Supply})P_i = P_i(x_{1i} + \cdots + x_{ni}) - P_i x_i,
$$

and differentiating with respect to $P_i$. Elasticities can then be formed by multiplying and dividing by $X_i$, leaving the typical term $\partial B_i/\partial P_i = X_i(e_{ij} - e_{ij})$. (This assumes that units are chosen so that each $P_i$ is initially unity.)

\(^{27}\) Cournot made extensive use of the proposition that the sum of all balances is necessarily zero [2, Ch. 3].
where
\[ \Delta = \begin{vmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{vmatrix} \]
and where \( \Delta_{ji} \) is the cofactor of the \( j \)th row and the \( i \)th column of \( \Delta \).

Equations (42) provide a general framework into which specific policy changes can be introduced. But in order to evaluate any of the signs, \( dP_i/d\alpha \), it is necessary to know the values, or at least the signs, of two kinds of terms: the coefficients \( \partial B_j/\partial \alpha \) and the ratios \( \Delta_{ji}/\Delta \). Now the coefficients \( \partial B_j/\partial \alpha \) describe the change in the balances of payments arising from the policy change at constant prices: to evaluate these coefficients, then, we can apply the method of comparative statics. The ratios \( \Delta_{ji}/\Delta \), on the other hand, indicate the interactions of price changes in multiple markets, and the effectiveness of price changes in relieving the initial disequilibrium. In the two-country case these signs were determined by the stability conditions; in the multiple-country case, however, it is easily shown that some of the ratios may be positive while others are negative without conflicting with the conditions of stability. It appears, then, that in evaluating equations (42) we will be left with some positive and some negative terms, and no general presumption about the sign of \( dP_i/d\alpha \).

To make progress some restriction on the signs of the elements \( b_{ij} \) in the basic determinant \( \Delta \) is required. The most interesting special case, for present purposes, is that where \( b_{ij} > 0 \) for \( i \neq j \). This assumption means that an increase in the price of the exports of any country, other prices being held constant, improves the balance of payments of every other country; it also implies, by Cournot’s Law, that a rise in the price level in one country worsens that country’s balance of payments. From this assumption flow two important deductions: (1) the system is stable under the usual dynamic postulates\(^{28} \) [1][4][20];

\(^{28}\) There are two approaches to the stability of international equilibrium. One approach is to treat the world economy like the domestic economy and postulate that the price of any good rises and falls in proportion to the excess demand and supply of that good. For example, the dynamic system may be written as follows:

\[ \frac{dP_i}{dt} = k_r \sum_{s=1}^{m} (\eta_{rs} - X_{rs}) \]

where the summation is over countries. By linearizing (1) (retaining only linear terms of a Taylor series), and translating the resulting partial derivatives into demand and supply elasticities, we obtain:

\[ \frac{dP_i}{dt} = k_rX_r \sum_{q=1}^{m} (\eta_{rq} - \epsilon_{rq})(P_q - P_q^b) \]

where the own elasticities of demand (\( \eta_{r} \)) and supply (\( \epsilon_{r} \)) in the world as a whole are defined to be, normally, negative and positive respectively. The linear system can be stable only if the real parts of the roots of the following equation:

\[ k_rX_r(\eta_{r} - \epsilon_{r}) - \delta_{r}\lambda = 0 \]

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Productivity Changes. Suppose that output and expenditure in country 0, the numeraire country, increase by \( dX_0^* \). At constant prices, assuming no inferior goods, inhabitants of country 0 buy more of all goods, creating a deficit in their own balance and a surplus in the balance of every other country. The surplus in the balance of country \( i \) is \( m_{i0}dX_0^* \) where \( m_{i0} \) is the marginal propensity to spend in country 0 on the goods of the \( i \)th country. The typical term in equation (42) is therefore

\[
- B_j \frac{\partial B_j}{\partial \alpha} = - m_{j0}.
\]

are all negative (\( \delta_{ij} \) is the Kronecker delta). The theorem on gross substitutes states that if all cross elasticities are positive (including the cross elasticities of demand for the numeraire good) the system is stable.

The above system focuses attention on world markets for particular commodities. Classical international trade theorists on the other hand (with the exception of F. D. Graham) emphasized the importance of disequilibrium in the balance of payments, gold flows and changes in the terms of trade. If we now let \( P_i \) denote the world price of the exports of country \( i \) we have the following system which is more compatible with the postulates of classical theory:

\[
\frac{dP_i}{dt} = h_iB_i(P_1, \ldots, P_n) \quad (i = 1, \ldots, n)
\]

assuming that there are now \( n+1 \) countries, and that prices are expressed in terms of the exports of country 0. Following the same procedure as above we find that stability requires that the real parts of the roots of the following equation:

\[
| h_i b_{ij} - \delta_{ij} \lambda | = 0
\]

must all be negative. Again, stability is assured if all \( b_{ij} > 0 \) for \( i \neq j \). If prices were all constant and exchange rates were all flexible the system would be stable if all currencies were gross substitutes.

Generally the systems (1) and (4) are fundamentally different. Goods may all be gross substitutes while some currencies are gross complements, and vice versa. The gap between the two systems narrows, however, when only one country produces each good: In that case an excess world demand for a good implies an excess demand for the currency of the country producing that good.

I have said that the system (4) conforms more closely to the classical system than does system (1). This is not meant to imply that classical theorists would accept even as an approximation the rigid dynamic laws postulated. Consider, for example, the following passage from Marshall's privately circulated manuscript of 1879 [11, pp. 19, 25]:

\[
\ldots \text{so that if we chose to assign to these horizontal and vertical forces any particular laws, we should obtain a differential equation for the motion of the exchange index} \ldots \text{Such calculations might afford considerable scope to the ingenuity of those who devise mathematical problems, but} \ldots \text{they would afford no aid to the economist.}
\]

For the mathematical functions introduced into the original differential equations could not, in the present condition of our knowledge, be chosen so as to represent even approximately the economic forces which actually operate in the world. \ldots\] Whereas the use of mathematical analysis has been found to tempt men to expend their energy on the elaboration of minute and complex hypotheses, which have indeed some distant analogy to economic conditions, but which cannot properly be said to represent in any way economic laws.
By substitution, then, we get the criterion for the change in the terms of trade of the growing country

\[
\frac{dP_i}{dX_i} = - \sum_{j=1}^{n} m_{ji} \frac{\Delta_{ji}}{\Delta}.
\]

Every \(m_{ji}\) is positive in the absence of inferior goods, and every \(\Delta_{ji}/\Delta\) is negative if all exports are gross substitutes. Therefore an improvement in productivity unambiguously worsens the commodity terms of trade of the growing country: the prices of the exports of every other country rise relative to the price of the exports of the growing country.\(^{29}\)

**Tariff Changes.** Suppose that country 0 applies an undiscriminatory tariff equal to \(d_t\) on all imports. At constant foreign prices the tariff-inclusive price of all imports in country 0 rises by \(d_t\). On the assumption that the government spends the tariff proceeds on home goods, the excess demand for the goods of typical country \(j\), at constant foreign

\[^{29}\]Why has country 0, whose exports are numeraire, been chosen as the growing country? Suppose instead that country \(i\) grows by \(dX_i\). Then substituting in equation (42) the terms

\[
\frac{\partial B_i}{\partial X_i} = m_i
\]

(the aggregate marginal propensity to import in country \(i\)), and the terms

\[
\frac{\partial B_j}{\partial X_i} = -m_{ji},
\]

when \(i \neq j\), we obtain the criterion:

\[
\frac{dP_i}{dX_i} = -m_i \frac{\Delta_{ii}}{\Delta} - \cdots + m_i \frac{\Delta_{ii}}{\Delta} - \cdots - m_{ii} \frac{\Delta_{ii}}{\Delta}.
\]

But \(m_i \Delta_{ii}/\Delta\) is negative while all the other terms are positive. It would therefore appear that no unambiguous result is possible, and that the method of treating the change as occurring in the numeraire country is a special case.

Nevertheless the economist's intuition tells him that in static analysis the choice of numeraire cannot affect the ultimate change in relative prices; and that if the terms of trade of the numeraire country deteriorate when that country grows, the terms of trade of any other country must fall when it grows. He may therefore conclude that \(P_i\) must fall in equation (1), that the negative term dominates.

This is in fact correct. Making use of the definition of the aggregate marginal propensity to import, i.e., \(m_i = m_{ii} + m_{ij} + \cdots + m_{ni}\), we can rearrange (1) to get:

\[
\frac{dP_i}{dX_i} = m_{0i} \frac{\Delta_{ii}}{\Delta} + m_{ii} \frac{\Delta_{ii} - \Delta_{ii}}{\Delta} + \cdots + m_{ni} \frac{\Delta_{ii} - \Delta_{ii}}{\Delta}.
\]

Now the first term on the right of (2) is clearly negative. The other terms will be negative if the principal cofactor of \(\Delta\) dominates each of the other cofactors, i.e., if \(|\Delta_{ii}| > |\Delta_{ij}| (j \neq i)\). But by subtraction of the two cofactors it is easily shown that the resulting \((n-1)\)th order determinant \(\Delta_{ii} - \Delta_{ii}\) has all the characteristics of \(\Delta\) (positive off-diagonal elements and dominant negative diagonal elements) except that its sign is opposite to the sign of \(\Delta\). (An analogous theorem has been proved by Metzler in analysis of the matrix multiplier.) All the terms on the right of (2) are therefore negative, so \(P_i\) must fall as a result of growth in country \(i\).

To avoid these complications I have, in the text, supposed that the policy change occurs in the country whose exports are numeraire, though the result is perfectly general. The same applies to the analysis of tax and tariff changes analyzed below.
prices, is \(-\eta_{j0}I_{j0}dt_0\), where \(\eta_{j0}\) and \(I_{j0}\) are, respectively, the elasticity of demand for imports (with respect to own price) and the level of imports, from country \(j\) to country 0. The typical term from the general equation (42) becomes

\[ \frac{-dB_i}{dI_0} = \eta_{j0}I_{j0}. \]

The criterion for the change in the terms of trade of country 0 is therefore:

\[ \frac{dP_i}{dI_0} = \sum_{j=1}^{n} \eta_{j0}I_{j0} \frac{\Delta_{ji}}{\Delta}. \]

The elasticities are all defined to be positive (Giffen goods are ruled out by the assumption of gross substitution) so the conclusion is again unambiguous: An increase in tariffs raises the price of the exports of the tariff-imposing country relative to the prices of the exports of all other countries, on the assumption that tariff proceeds are spent on home goods.30

**Consumption and Production Tax Changes.** The effects of a consumption tax on import goods are equivalent to the effects of a tariff if there is no domestic production of these goods. If there is domestic production of import goods then the typical term, from the skeleton equation (42), is \(\eta_{j0}y_{j0}\), where \(\eta_{j0}\) is the elasticity of consumer demand in 0 for the products exported by country \(j\), and \(y_{j0}\) is the level of domestic consumption of these products. This assumes again that tax proceeds are spent on domestic goods. The criterion for the change in the relative price of the exports of the typical country \(i\) is therefore:

\[ \frac{dP_i}{dI_0} = \sum_{j=1}^{n} \eta_{j0}I_{j0} \frac{\Delta_{ji}}{\Delta}, \]

which makes use of the relation between net and gross elasticities, \(\eta_{j0} = \eta_{j0} + m_{j0}\). Notice that the sign of \(dP_i/dI_0\) is not unambiguously determined; in both equations the coefficients of the cofactors are composed of one positive and one negative term, and we have no information about which term is dominant. It is therefore possible that some prices rise relative to the price of the exports of the tariff-imposing country.

This indefinite result differs from the two-country analysis where the net effect of the tariff is a pure substitution effect away from imports and to home goods. On the assumption that all exports are gross substitutes, and that no goods are inferior, all substitution effects are positive in the multiple-country analysis also; but it cannot, as far as I know, be proved that the net effect of the tariff is a shift of demand away from the imports of every country. A tariff applied to the exports of a particular country creates a deficit in that country’s balance of payments equal to a pure substitution effect; but if, in addition, tariffs are applied to the exports of third countries, the income effect resulting from the distribution of the tariff proceeds of the third countries to some extent offsets or reverses the initial deficit in the first country’s balance.

It should be possible to prove, however, that at least some foreign prices fall relative to the price of the exports of the tariff-imposing country, by making use of the proposition that net substitution must be away from imports and to home goods.

30 If the tariff proceeds are redistributed to consumers an income effect must be added to each term in (44). The tariff proceeds are equal to \(I_0dI_0\), where \(I_0\) is the initial aggregate level of imports, so the typical income term is \(-m_{j0}I_0\). The criterion then becomes:

\[ \frac{dP_i}{dI_0} = \sum_{j=1}^{n} \left[ \eta_{j0}I_{j0} - m_{j0}I_0 \right] \frac{\Delta_{ji}}{\Delta} = \sum_{j=1}^{n} \left[ \eta_{j0}I_{j0} - m_{j0}(I_0 - I_{j0}) \right] \frac{\Delta_{ji}}{\Delta}, \]

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which is necessarily negative. Thus the terms of trade improve with respect to all countries as a result of a tax on imported goods. By similar analysis it can be shown that the terms of trade worsen as a result of an increased tax on the consumption of export goods. And by changing signs we get the criteria for the effects of subsidies on the consumption of import and export goods.

By similar analysis it can be shown that the typical coefficient of (42) resulting from a tax on the production of exportables is

\[ \frac{dP_i}{dP_0} = \sum_{j=1}^{n} \eta_{ij} y_{j0} \frac{\Delta_{ij}}{\Delta} \]

Thus if supply elasticities are all positive a tax on the production of exportables improves the terms of trade of the taxing country with respect to all other countries. [Note that each term in (46) is zero if country 0 is completely specialized.] The same criterion applies, mutatis mutandis, for production taxes on imported goods, or production subsidies. (See Figure 8.)

Unilateral Payments. Suppose that country 0 pays country S an annual tribute, or gift, or loan (ignoring interest payments). Then expenditure in 0 decreases and in S decreases by the amount of the payment. The demand for all goods in 0 decreases and in S increases if there are no inferior goods. At constant prices the deficit created by these expenditure changes in the balance of payments of the jth country is

\[ (m_{j0} - m_{j0})dT_{s0} \]

where \( dT_{s0} \) is the value of the transfer. The criterion for the change in the price of exports in the receiving country, relative to the price of the exports of the paying country, is therefore:

\[ \frac{dP_s}{dT_{s0}} = \sum_{j=1}^{n} (m_{j0} - m_{js}) \frac{\Delta_{js}}{\Delta}. \]

[In the term \( j = s \) the coefficient is \( (m_{s0} - m_{ss}) = (m_{s0} - c_s) = (m_{s0} + m_s - 1) \) where \( c_s \) and \( m_s \) are, respectively, the marginal propensities to consume and import in country S.] The unilateral payment, through expenditure changes in the transferor and transferee, rearranges demand throughout the world in a way which does not permit any a priori generalization, a result which can be expected from the analysis of transfer between two countries.\(^3\)

\(^3\) The propositions in this section require modification if there are limiting cases such as infinite or zero elasticities, and zero or negative marginal propensities to spend.
Assume three countries, the U.S. (0), the U.K. (1) and France (2) with the respective currency prices (expressed in dollars) $P_1$, $P_2$ and $P_3$. Choose units of pounds and francs so that, at equilibrium, $P_1$ and $P_2$ are each unity. (If equilibrium ratios are $1 = 1/2.80$ pounds = 5 francs then 1/2.80 pounds and 5 francs become the British and French currency units.) The two curves $B_1B_1$ and $B_2B_2$ trace the loci of pound-franc prices which allow equilibrium in the British and French balances. At Q both balances are in equilibrium implying, by Cournot's Law, equilibrium in the U.S. balance.

If all currencies are gross substitutes both curves have positive slopes: this follows because appreciation of the franc must be associated with appreciation of the pound to maintain equilibrium. Moreover, $B_1B_1$ has a slope greater than 45 degrees and $B_2B_2$ has a slope less than 45 degrees; this follows because of our choice of currency units and because appreciation of the dollar is equivalent to depreciation of the franc and pound in equal proportion; a movement along the line OQ from Q to Q' must improve the French and British balances and worsen the U.S. balance. Four quadrants can then be identified: east and west of $B_1B_1$ there are deficits and surpluses, respectively, in the British balance; and north and south of $B_2B_2$ there are deficits and surpluses in the French balance.

On the dynamic assumption that the dollar prices of pounds and francs rise and fall in proportion to the disequilibrium in the British and French balances, the arrows in each quadrant indicate the forces impelling a return to equilibrium. The reader may easily satisfy himself that the equilibrium position, once disturbed, will be restored. From the disequilibrium position A, for example, the path may follow the broken line AQ, becoming "trapped" in Quadrant III and hence move to equilibrium.

To determine geometrically the movement of the prices as a result of policy changes it is necessary to indicate the direction in which the two curves shift. If U.S. output and expenditure increase, more British and French goods are demanded at constant prices improving both British and French balances; the two curves therefore shift away from the origin and the new equilibrium point moves to somewhere in Quadrant I. Similarly, for the tariff and tax changes analyzed in the text both curves shift toward the origin with an unambiguous improvement in the U.S. terms of trade.
One special case, however, is of interest in view of recent discussions of foreign aid and "tied" exports. Suppose that country 0 makes a gift to a foreign country but requires that the gift be spent on its own exports. (It may be supposed that the gift is financed by taxation and that the financial transfer involves the grant of credits from an export-import bank.) In this case the price of the exports of the paying country rises relative to the price of the exports of the receiving country (all the terms $m_{ij}$ are zero). This does not imply, however, that the transfer "burden" is negative—that real income in the transferring country declines by less than the gift—since the prices of the exports of some of the third countries may rise. It further assumes that the receiving country does not re-export the tied exports to third countries.

Comparison of Two-Country and Multiple-Country Models. The foregoing account of policy changes in a system containing many national units naturally leads to more complicated conclusions than in the case of a system containing only two countries. Applying only the restrictions of stability, I have not been able to show that two-country and multiple-country systems lead to substantially the same conclusions. The difficulty lies in relations of gross complementarity among the exports of the various countries. Gross complementarity is consistent with stability in the multiple-country system but inconsistent with stability in the two-country system. The two-country model cannot therefore be expected to suggest "theorems which admit of extension to more concrete cases" when gross complementarity is involved.

However, when gross complementarity is absent—when all exports are gross substitutes—there is a remarkable similarity between the conclusions of the two models. Productivity, tariff and tax changes move the terms of trade in the same direction in the many-country system as in the simpler system. The explanation of this similarity lies in what may be called the Law of Composition of Countries. If all foreign exports are perfect substitutes for each other, the foreign countries can be aggregated into a composite country and called the "rest of the world"; in that case the conclusions of the two models will agree both qualitatively and quantitatively. But if foreign exports are only imperfect substitutes for each other exact results cannot be obtained by the use of a composite country. However, while the quantitative conclusions of the two models will in this case differ, the qualitative conclusions, with one qualification, remain. There is a presumption, then, that the use of two-country models will not be subject to serious error provided that all exports are gross substitutes.

32 The qualification refers to the treatment of income effects examined in footnote 31.

The most important generalization of the classical system has been provided by Mosak [18].