# Real Money Balances: An Omitted Variable from the Production Function?

Allen Sinai; Houston H. Stokes

*The Review of Economics and Statistics* is currently published by The MIT Press.

# REAL MONEY BALANCES: AN OMITTED VARIABLE FROM THE PRODUCTION FUNCTION?

Allen Sinai and Houston H. Stokes *

## I Introduction

SEVERAL writers have argued that real money balances are a factor of production.[1] No one, however, has directly tested the hypothesis that real money balances are a factor input.[2] The purpose of our paper is to report the results of such a test. We find that real money balances, regardless of definition, enter significantly in a Cobb-Douglas production function fitted to annual data over the period 1929–1967 for the private domestic sector of the United States economy. Quantity indices of output, capital and labor, published by Christensen and Jorgenson (1970) and adjusted both for quality changes and rates of utilization, are employed to estimate the production function. Data for nominal money balances are taken from Friedman and Schwartz (1970). Our results have important implications for production function analysis, the explanation of total factor productivity, and monetary growth theory.

[1] Bailey (1971, pp. 54–56), Friedman (1959, p. 334), (1969, p. 14), Johnson (1967, pp. 40–41), and Nadiri (1970, p. 1153, fn. 27) (1969, p. 175) assert that money belongs in the production function. Levhari and Patinkin (1968, pp. 737–740) analyze the growth of per capita output in a neoclassical model where money balances are assumed to be a producer's good. They criticize the neoclassical monetary growth model of Tobin (1965) for failing to consider the role of money balances as a consumer's good or as a factor of production.

[2] Nadiri (1969, p. 174) derived a demand for cash balances in the manufacturing sector under the assumption that real balances are in the production function. He found that the demand for cash balances depended on output, the interest rate, expected changes in price and relative factor prices. His results *indirectly* support the hypothesis that cash balances are a factor of production.

The plan of the paper is as follows. Section II deals with a brief discussion of the rationale for the presence of real balances in the production function. In section III we present the production function used in the study and discuss the data employed. Results are given in section IV. A summary and conclusions follow in the final section.

## II Real Money Balances and Production

The rationale for including real money balances in the production function relates, in part, to the increased "economic efficiency" of a monetary economy compared with a barter economy. The standard neoclassical production function is concerned with real output and real inputs. Yet, in order to obtain inputs, to organize and combine them in production, it is necessary to engage in exchange. In the barter economy which implicitly underlies traditional production functions, considerable labor and capital may be expended to insure whatever trade is necessary for viable economy wide production. Labor must be hired, capital services rented, and factors combined at various physical locations so that production can occur. Laborers, owners of capital goods and entrepreneurs must go to markets and exchange physical goods and services in return for services and goods. Time, effort and capital may be utilized in the process. Thus, labor and capital may be diverted from production to distribution in order to achieve the "double coincidence" required in a barter economy. However, with money entering as a medium of exchange the search for acceptable "terms of trade" can be avoided. In a monetary economy, productive efficiency may increase as labor and capital services, released from the special tasks required in a barter economy, are used in production.

If firms hold money balances they must anticipate a return from doing so. Why else would they hold them? Stein (1970, p. 90) suggests that the expected return on real bal-

ances has three components — a marginal product, anticipated appreciation (depreciation) in the command over goods and services, and a "liquidity" yield reflecting the opportunity cost of owning an asset which fluctuates less than other assets in real value. The marginal product of money balances is the increased output obtained as a consequence of increases in real balances which release additional labor and capital services for utilization in production instead of distribution. Anticipated appreciation of the command over goods and services depends on expected price changes. The liquidity yield is affected by movements in expected prices of financial assets.

There are numerous implications of real money balances as a factor of production. First, money would have a marginal physical productivity schedule like other inputs. Second, firms' demands for real balances would be derived in the same way as other factor demand functions.[3] Third, changes in the stock of money would affect real output, contrary to the classical dichotomy which implies the neutrality of money. Fourth, real balances might explain some of the rate of growth of total factor productivity or the "residual."[4] Finally, traditional analyses of production would be subject to modification.

### III Model and Data

The hypothesized production function was Cobb-Douglas with nonconstant returns to scale, i.e.,

$$Q = Ae^{\lambda T} L^a K^\beta m^\gamma u \qquad (1)$$

where

$Q$ = output
$L$ = labor
$K$ = capital
$m$ = real money balances
$T$ = time
$A$ = efficiency parameter
$\lambda$ = rate of disembodied or neutral technological change
$a$ = elasticity of output with respect to labor
$\beta$ = elasticity of output with respect to capital

[3] See, e.g., Nadiri (1969, pp. 175–180).
[4] There has been considerable disagreement over the source(s) of changes in total factor productivity. Technological change, errors in the measurement of factor inputs, or misspecification of the production function are possible candidates. See Nadiri (1970, pp. 1140 ff.).

$\gamma$ = elasticity of output with respect to real money balances
$u$ = disturbance term.

The Cobb-Douglas specification was selected because in an exploratory study it seemed more appropriate to begin with a relatively simple and widely used function.[5] Furthermore, the "correct" specification of the production function is uncertain despite the extraordinary number of investigations on the subject. Substantial differences in parameter estimates of alternative functional forms have been obtained in cross-section and time-series studies, in studies concerned with different time periods or at various levels of aggregation, and where alternative definitions of variables have been employed. We did not want to become involved in difficult problems of specification and estimation when our purpose was simply to examine the potential significance of real money balances in the production function. For similar reasons neutral technological progress was assumed.

Equation (1) was estimated in log linear form as

$$\ln Q = \ln A + a\ln L + \beta\ln K + \gamma\ln m + \lambda T + u'. \qquad (2)$$

Ordinary least squares was employed initially

[5] To evaluate the validity of our assumed functional form against one possible alternative, the CES production function, we applied a test suggested by Kmenta (1967, pp. 180–181). The results of this test indicated the appropriateness of the Cobb-Douglas hypothesis for our data and for the period covered by our study. The results were

$$\ln Q = -3.727 + 1.801 \ln L + .467 \ln (K/L)$$
$$\quad (.247) \quad (.040) \quad\quad (.241)$$
$$+ .056 [\ln (K/L)]^2$$
$$\quad (.235)$$

$$\bar{R}^2 = .995$$
$$\text{S.E.E.} = .033$$
$$\text{D.W.} = 1.50$$

where $\bar{R}^2$ is the adjusted coefficient of multiple determination, S.E.E. is the standard error of estimate, and D.W. is the Durbin-Watson statistic. The coefficient of the last term in the regression was not statistically significant. It is equal to $-\frac{1}{2}\rho\gamma\delta$ where $\rho$ is a substitution parameter, $\gamma$ a scale parameter, and $\delta$ a distribution coefficient. The coefficient of $\ln (K/L)$ was significant and is defined as $\gamma\delta$. Therefore $\rho$ equals zero and $\sigma = 1/(1+\rho)$, the elasticity of substitution, is unity. Interestingly, $\gamma$ was 1.801 which indicates increasing returns to scale. In equation (1) we assumed nonconstant returns. In our subsequent empirical work we found increasing returns to scale (with and without real money balances in the production function) of the order of magnitude in the Kmenta approximation, e.g., see table 1. When a time trend was added to our estimating equations (see table 2) the returns to scale fell, although remaining above unity, indicating a possible effect of neutral technological progress on productivity.

but in cases where the residuals were serially correlated a correction for autocorrelation was applied.[6] Problems of simultaneous equations bias and aggregation, although potentially substantial, were not dealt with at this time.

Data for output, labor, and capital were taken from a recent study by Christensen and Jorgenson (1970). They constructed Divisia quantity indexes of gross private domestic product, labor and capital services.[7,8] The major advantage in using their data was that measures of input were corrected for quality change and rates of utilization.[9] Failure to employ service-in-use measures of inputs with "embodied technological change" somehow accounted for, would have resulted in biased estimates of regression coefficients.[10]

Data on nominal money balances were obtained from Friedman and Schwartz (1970, pp. 24–72). Conventional definitions of money, $M1$ and $M2$, were employed. An additional measure of nominal money, $M3$, also was tried. This latter concept is defined as $M2$ plus deposits at Mutual Savings Banks and Postal Savings Systems. The money stock figures were deflated by a Divisia index of factor prices prepared by Christensen and Jorgenson (1970, pp. 35–37). Our measures of real money balances thus represented real purchasing power over

TABLE 1. — ESTIMATES OF THE PARAMETERS OF THE COBB-DOUGLAS PRODUCTION FUNCTION, WITH AND WITHOUT REAL MONEY BALANCES, WITH A TIME TREND, 1929–1967

| $\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T + u'$ | | | | | |
| --- | --- | --- | --- | --- | --- |
| | Equation Number | | | | |
| | (3) | (4) with $m1$ | (5) with $m2$ | (6) with $m3$ | (7) with $T$ |
| $\ln A$ | −3.640 | −3.022 | −3.537 | −3.820 | −2.032 |
| | (.250) | (.264) | (.250) | (.276) | (.603) |
| $A$ | .026 | .049 | .029 | .022 | .131 |
| $\alpha$ | 1.356 | .945 | 1.092 | 1.194 | 1.195 |
| | (.087) | (.123) | (.101) | (.095) | (.097) |
| $\beta$ | .428 | .585 | .470 | .429 | .234 |
| | (.050) | (.058) | (.049) | (.050) | (.082) |
| $\gamma$ | | .172 | .214 | .194 | |
| | | (.045) | (.061) | (.069) | |
| $\alpha+\beta+\gamma$ | 1.784 | 1.702 | 1.776 | 1.817 | 1.429 |
| $\lambda$ | | | | | .010 |
| | | | | | (.003) |
| $\bar{R}^2$ [a] (orig.) | .9943 | .9951 | .9947 | .9945 | .9951 |
| S.E.E. [b] (orig.) | .0347 | .0326 | .0338 | .0343 | .0327 |
| $\bar{R}^2$ | .994 | .993 | .993 | .994 | .994 |
| S.E.E. | .036 | .035 | .037 | .037 | .033 |
| D.W. [c] | 1.54 | 1.43 | 1.33 | 1.25 | 1.44 |

$\ln Q$ = natural log gross private domestic product, quantity index. $\ln L$ = natural log private domestic labor input, quantity index. $\ln K$ = natural log private domestic capital input, quantity index. $\ln m$ = natural log real money balances, $m1$, $m2$, $m3$. $T$ = time trend, 1929 = 0.

Standard errors of regression coefficients in parentheses.

[a] $R^2$ (orig.) = adj. coeffic. of determ. for equation *not* corrected for autocorrelation.  [b] S.E.E. (orig.) = std. error of est. for equation *not* corrected for autocorrelation.  [c] D.W. = Durbin-Watson statistic for corrected equations.

factor inputs. Since real balances are hypothesized to affect productivity by facilitating the exchange necessary to obtain factor inputs, deflating the nominal money stock by factor prices rather than by commodity prices seemed more appropriate.

## IV Results

Table 1 shows the results of estimating a Cobb-Douglas production function (equation (3)), Cobb-Douglas equations with real money balances (equations (4–6)) and a Cobb-Douglas function with a time trend (equation (7)). We only report results for regressions that are corrected for autocorrelation.[11]

Table 1 indicates that real money balances, whether defined as $m1$, $m2$, or $m3$, are of substantial importance when added to the stan-

---

[6] A generalized least squares technique was employed to correct for autocorrelation. The method utilized was the one outlined in Johnston (1963, pp. 179–187) except for a scale adjustment described in Thornber (1966). Failure to deal with the problem of autocorrelated residuals in the estimation of production functions has been recognized as harmful to parameter estimates. See, e.g., Bodkin and Klein (1967, p. 43). Walters (1963, p. 426) assumed a first order autocorrelation coefficient of 0.5, transformed data for 1909–1949, and re-estimated an unconstrained Cobb-Douglas production function by ordinary least squares. His results showed substantial differences in the standard errors of the coefficients estimated from the transformed data.

[7] For a discussion of the "nice" properties exhibited by Divisia indices, see Richter (1970).

[8] A complete description of the variable definitions, data and sources is given in the appendix.

[9] Christensen and Jorgenson (1970, pp. 28–35).

[10] To examine the effect of using national income series instead of the Christensen-Jorgenson data we reran all regressions with conventional measures of the variables. Substantial differences were exhibited in the size of the coefficients obtained from the alternative sets of data, especially for the coefficients of capital and real balances. These were considerably higher when national income data were used. The coefficients of real money balances, however defined, were still highly significant. Detailed tables of these results are available upon request from the authors.

[11] See footnote 8. Autocorrelation leads to underestimates of sampling variances of regression coefficients and invalid tests of significance. Second, the estimated residual variance, hence standard error of estimate, is likely to be biased downward. Therefore, we do not report equations where serial correlation is present.

dard Cobb-Douglas production function. Co-efficients of real balances are highly significant along with the coefficients of labor and capital. The real stock of money accounts for about 10–15 per cent of the returns to scale exhibited by the production function. The standard error of estimate is reduced in the equation with $m1$ but increases when $m2$ or $m3$ is used. However, goodness of fit comparisons are invalid between equations estimated by generalized least squares since data transformations in different regressions use different values for the autocorrelation coefficient. As a result the dependent and independent variables are not the same in each case. A better way to evaluate the relative goodness of fit is to compare the adjusted coefficients of multiple determination or standard errors of estimate for the regressions that are not corrected for autocorrelation. The former are 0.9943, 0.9951, 0.9947, and 0.9945 for equations (3–6), respectively, indicating that real money balances reduces the unexplained variance of gross private domestic output.

The high degree of increasing returns to scale exhibited in equations (3–6) may have been due to the omission of an appropriate variable for neutral technological progress. Without a separate measure of technology the regression coefficients would be biased upward. Results obtained by adding a time trend to equations (3–6) lend support to this view. Returns to scale fall from 1.784 in the standard Cobb-Douglas formulation to 1.429 in equation (7). The addition of a trend to the equations with real money balances (see table 2) is associated with a drop in returns to scale from a range of approximately 1.7–1.8 to 1.2–1.6.

The finding of increasing returns to scale in the aggregate production function is similar to results obtained by Bodkin and Klein (1967). In single equation estimation of a Cobb-Douglas production function containing a time trend, the sum of the coefficients of labor and capital was 1.202 (1967, p. 36, table 1). Simultaneous equation estimation of the production function and a marginal productivity condition representing cost minimization gave a returns to scale coefficient of 1.456 (1967, p. 36, table 2). It is interesting to note that in our table 2 the returns to scale are 1.521, 1.556, and 1.537

TABLE 2. — ESTIMATES OF THE PARAMETERS OF THE COBB-DOUGLAS PRODUCTION FUNCTION, WITH REAL MONEY BALANCES AND TIME TREND, 1929–1967

$\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T + u'$

| | Equation Number | | |
| | (8) with $m1,T$ | (9) with $m2,T$ | (10) with $m3,T$ |
|---|---|---|---|
| $\ln A$ | −2.273 | −2.574 | −2.545 |
| | (.600) | (.691) | (.759) |
| $A$ | .103 | .076 | .078 |
| $\alpha$ | .966 | 1.100 | 1.174 |
| | (.123) | (.104) | (.099) |
| $\beta$ | .428 | .323 | .276 |
| | (.116) | (.098) | (.090) |
| $\gamma$ | .127 | .133 | .087 |
| | (.051) | (.072) | (.079) |
| $\alpha+\beta+\gamma$ | 1.221 | 1.556 | 1.537 |
| $\lambda$ | .006 | .006 | .008 |
| | (.004) | (.004) | (.004) |
| $\bar{R}^2$ [a] (orig.) | .995 | .995 | .995 |
| S.E.E. [b] (orig.) | .0325 | .0331 | .0332 |
| $\bar{R}^2$ [a] | .995 | .995 | .995 |
| S.E.E. [b] | .0329 | .0332 | .0327 |
| D.W. [c] | 1.45 | 1.32 | 1.31 |

$\ln Q$ = natural log gross private domestic product, quantity index. $\ln L$ = natural log private domestic labor input, quantity index. $\ln K$ = natural log private domestic capital input, quantity index. $\ln m$ = natural log real money balances, $m1$, $m2$, $m3$. $T$ = time trend, 1929 = 0.

Standard errors of regression coefficients are in parentheses.

[a] $\bar{R}^2$ (orig.) = adjusted coefficient of determination for equation *not* corrected for autocorrelation. [b] S.E.E. (orig.) = standard error of estimate for equation *not* corrected for autocorrelation. [c] D.W. = Durbin-Watson statistic for corrected equation.

for equations (8–10), respectively. The Bodkin-Klein study covered the period 1909–1949 and used data relating to the private domestic sector. They concluded, "It is our belief that the American economy does function under increasing returns to scale, and, as indicated in section III, there are other studies to support this contention" (1967, p. 42). Our results also support their argument, perhaps more strongly than their own work since we cover a more recent period and utilize superior measures of inputs.[12]

Adding real money balances to the standard Cobb-Douglas formulation appeared to primarily affect the coefficient of labor which fell from 12–30 per cent in equations (4–6). The degree of returns to scale remained approximately the same. This is what we might expect

[12] The measure of capital used by Bodkin and Klein was not corrected for either quality change or the rate of utilization. In their single equation estimation of the Cobb-Douglas function, the coefficient of capital stock was not statistically significant. A man-hours measure was used by them for labor whereas the Christensen-Jorgenson labor input series included corrections for intensity of use and changes in the composition of the labor force due to educational attainment.

since labor services are most likely to be released from distribution activities when real money balances increase. In the equations containing real balances the contribution of labor and capital to the total for both of them was about two-thirds and one-third, respectively, which is what we have come to expect on the basis of a number of studies that have assumed constant returns to scale. However, in the regressions without real balances, the contribution of labor was much higher. For example, in equation (3), labor's share of the total returns to scale was 76 per cent while in equation (7) it was 83.6 per cent.[13]

The marginal product of real balances implied by the coefficient of $m1$, 0.172, is 0.445.[14] This means that a dollar's increase in purchasing power over capital and labor is associated with about a four-tenths unit increase in real output for the private sector. Marginal products of 0.381 and 0.293 are implied by the regression coefficients for $m2$ and $m3$. These magnitudes do not seem unreasonable and are high enough to suggest a return to holding money balances that is consistent with observed firm behavior.

In table 2 we show the results of adding a time trend to equations (4–6). The purpose of doing so was two-fold. First, the trend was a possible representation of neutral technological progress. Second, real money balances may have been a proxy for another excluded variable which varied systematically with time.

In two of three regressions, i.e., equations (8–9), the trend coefficient was not significant. The values of the coefficients of capital and real balances fell, but remained highly significant. The goodness of fit differed little from the corresponding equations in table 1 that did not contain the time trend. These results

suggest that the significance of the trend in equation (7), where real money balances were excluded, was due to its proxying for the effect of money in the production function. We conclude that neutral technological progress was not significantly different from zero over the period 1929–1967 and that the effect ascribed to technology in other studies has been overstated. Appropriate specification of the production function to include real money balances and proper definition of factor inputs to account for quality changes and utilization rates appears to eliminate the "residual."

## V Summary and Conclusions

In this paper we test the hypothesis that real money balances have been mistakenly omitted from the production function. The rationale for real balances as a factor input relates to its role in facilitating transactions, exchange and specialization, thus contributing to increased productivity. Numerous writers have asserted that real money balances are a producer's good but previously there has been no *direct* test of this assertion.

Our results indicate that real money balances, however defined, are an important input in the production function. Coefficients of real balances are significant at the 1 per cent level in equations where the real stock of money is added to a traditional Cobb-Douglas form. Coefficients of labor services and capital services remain highly significant in the presence of real balances. Addition of a time trend to a Cobb-Douglas function containing money balances does not affect the significance of the coefficients of labor, capital and real balances. The trend variable is significant in a regression without money balances but loses its significance when real balances are included. These results show real money balances to have a significant independent effect on productivity. The evidence also suggests the trend variable, hypothesized to represent neutral technological progress, may be a proxy for real money balances.

The Cobb-Douglas functions we estimate exhibit increasing returns to scale, a result that is consistent with Bodkin and Klein's (1967) estimates of the Cobb-Douglas for the period

[13] The relatively high elasticities of output with respect to labor are probably due to simultaneous equation bias. Bodkin and Klein (1967, p. 36) obtained a fairly substantial reduction from their single equation estimate of the labor coefficient in an unconstrained Cobb-Douglas function when they used a simultaneous equation technique. However, their coefficient of labor input was still 0.960. Our results would suggest a further reduction if real money balances had been in their equation.

[14] Calculated at the means of real gross private domestic product, from Christensen and Jorgenson (1970, tables 2 and 3) and real money balances, from Friedman and Schwartz (1970, pp. 24–72).

1909–1949. The returns to scale change little when real balances are added to the production function. The coefficient of capital services is hardly affected by the presence of money, however the labor service coefficient falls anywhere from 12 to 30 per cent, depending on the measure of real balances employed. Real balances and labor thus seem to be "complementary" in the sense that the sum of their contributions is approximately equal to the coefficient of labor services in the production function without money balances.

The results of our study have far-reaching implications. A number of theoretical and empirical issues are raised. First, what is the mechanism by which increases in real balances enhance productivity? Is there a formal theory of the effect of real balances on production that goes beyond the rationale offered in section II? It is one thing to *assert* real balances belong in the production function and another to formalize a theory. Just how could the central bank increase real balances? Real balances cannot be produced like labor or physical units of capital. In a closed economy the monetary authorities may be able only to control the nominal quantity of money. Under the assumption of full employment that is used in most monetary growth models it is hard to see how real money balances could be changed. A possible solution may be to define "full employment" in terms of exchange efficiency.

Second, if real balances are a factor input, traditional analyses of production have to be modified. Failure to include real balances may substantially affect estimates of production function parameters, e.g., the elasticity of substitution. Real balances could be a missing variable that has contributed to unexplained "residuals" being attributed to technological progress. Neoclassical analyses of factor demand also would be affected. Cash balances in the production function suggest a role for the internal financial position of the firm on the demand for labor or capital.

Finally, important evidence bearing on the appropriate model for monetary growth theory is provided. Stein (1970, p. 85, p. 105) has stated "equally plausible [monetary growth] models yield fundamentally different results. . . . The crucial question is: which is the cor-

rect monetary growth model?" Our results support the view that real money balances are a producer's good, an assumption made in some neoclassical monetary models of growth, e.g., in Levhari and Patinkin (1968).

## REFERENCES

Bailey, M. J., *National Income and the Price Level* (New York: McGraw-Hill Book Company, 1971).

Bodkin, R. G., and L. R. Klein, "Nonlinear Estimation of Aggregate Production Functions," this REVIEW, 49 (Feb. 1967), 29–45.

Christensen, L. R., and D. W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929–1967," *Review of Income and Wealth*, series 15 (Dec. 1969), 293–320.

——, "U.S. Real Product and Real Factor Input, 1929–1967," *Review of Income and Wealth*, series 16 (Mar. 1970), 19–50.

Friedman, M., "The Demand for Money: Some Theoretical and Empirical Results," *Journal of Political Economy*, 67 (Aug. 1959), 327–351.

——, "The Optimum Quantity of Money," in *The Optimum Quantity of Money and Other Essays* (Chicago: Aldine Publishing Company, 1969), 1–50.

——, and A. J. Schwartz, *Monetary Statistics of the United States* (New York: Columbia University Press, 1970).

Johnson, H. G., "Inside Money, Outside Money, Income, Wealth, and Welfare in Monetary Theory," *Journal of Money, Credit and Banking* (Feb. 1969), 30–45.

Johnston, J., *Econometric Methods* (New York: McGraw-Hill Book Company, 1963).

Kmenta, J., "On Estimation of the CES Production Function," *International Economic Review*, 8 (June 1967), 180–189.

Levhari, D., and D. Patinkin, "The Role of Money in A Simple Growth Model," *American Economic Review*, 58 (Sept. 1968), 713–754.

Nadiri, M. I., "Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey," *Journal of Economic Literature*, 8 (Dec. 1970), 1137–1178.

——, "The Determinants of Real Cash Balances in the U.S. Total Manufacturing Sector," *Quarterly Journal of Economics*, 83 (May 1969), 173–196.

Patinkin, D., *Money, Interest, and Prices* (New York: Harper and Row, Publishers, 1965).

Richter, M. K., "Invariance Axioms and Economic Indexes," *Econometrica*, 34 (Oct. 1966), 739–755.

Stein, J. L., "Monetary Growth Theory in Perspective," *American Economic Review*, 60 (Mar. 1970), 85–106.

Thornber, H., "A Stepwise Regression Program: A Manual for (BiMed 34T, 8 Mar. 1966)," Report no. 6603, Center for Mathematical Studies in Business and Economics, University of Chicago, March 1966.

Tobin, J., "Money and Economic Growth," *Econometrica*, 33 (Oct. 1965), 671–684.

Walters, A. A., "A Note on Economies of Scale," this REVIEW, 45 (Nov. 1963), 425–427.

# APPENDIX

## Definition of Variables and Sources of Data

*Output*: Defined as a Divisia quantity index number for gross private domestic product from the point of view of the producing sector. The source was Christensen and Jorgenson (1970, table 3, col. 1, pp. 30–31). Discussed in Christensen-Jorgenson (1970, pp. 20–23, 27–28).

*Labor*: Defined as a Divisia quantity index number for private domestic labor input. Labor input was measured by persons engaged and adjusted for effective hours per person and changes in the composition of the labor force by educational attainment. The source was Christensen-Jorgenson (1970, pp. 28–34).

*Capital*: Defined as a Divisia quantity index number for private domestic capital input. Capital input was measured by capital stocks for seven types of assets that were weighted by potential services prices and adjusted for relative utilization. The weighting by relative prices reflected "quality change" while relative utilization was based on the consumption of electricity relative to installed horsepower of electric motors. The

source was Christensen-Jorgenson (1970, table 5, col. 4, p. 36). Discussed in Christensen-Jorgenson (1970, pp. 34–37).

*Real Money Balances*: Defined as the money supply deflated by a Divisia quantity index of factor prices. The money supply was defined in three alternative ways. $M1$ was currency held by the public plus adjusted demand deposits of commercial banks. $M1$ was an annual average of seasonally adjusted quarterly totals and was in billions of dollars. $M2$ was $M1$ plus time deposits (adjusted) and was an annual average of quarterly totals (seasonally adjusted) in billions of dollars. $M3$ was $M2$ plus deposits at Mutual Savings Banks and Postal Savings Banks. $M3$ was an annual average of monthly totals (seasonally adjusted) and was in billions of dollars. The source for nominal $M1$, $M2$, and $M3$ was Friedman and Schwartz (1970, pp. 24–72). The source for the factor price index was Christensen-Jorgenson (1970, table 6, col. 2, p. 28).

*Time*: Defined as $T = 0$ in 1929 and numbered consecutively to 38 for 1967.