Measuring Expected Inflation; Further Tests in the Frequency Domain of a Proposed New Measure

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I. INTRODUCTION

In an ingenious article, Frankel (1982) proposed extracting a new measure of inflation from the interest rate term structure. His findings for the period 1959/8 to 1979/4 were that the market underestimated inflation in the 1970s and that his measure of expected inflation "does a slightly better job of predicting actual inflation, in terms of mean squared error, than do survey data." (Frankel (1882) page 140) This note provides a further test of the dynamic implications of the proposed new inflation measure by means of a test proposed by Geweke (1982a, 1982b, 1984).[1] It is argued that the Geweke procedure provides additional information on the dynamics of what is being measured in the proposed inflation measure beyond that contained in the mean squared error procedure utilized by Frankel,[2] which just related the new expected inflation measure and the actual price series at the same time period. An underlying assumption of the proposed testing procedure in this paper is that the appropriate way to measure the relationship between two series in the long run and the short run is in the frequency domain where short (long) run is high (low) frequency. The more usual way to relate series is in terms of lag length, with long (short) run being a long (short) lag length. The economic reason for using the frequency approach is that while the market place may be able to detect low frequency cycles, high frequency cycles, by their very nature, are harder to detect. Prior to a discussion of the Geweke procedure in detail and the statistical results of this paper in particular, it is important to relate the models proposed here to prior work on implicit expectations and rational expectations by Mills (1957) and Muth (1961), which have been summarized by Lovell (1986).

2. THE ECONOMETRIC CONSEQUENCES OF "IMPLICIT" AND "RATIONAL" EXPECTATIONS

If we assume that F_t is an expected (forecasted) inflation measure[3] and P_t is the actual inflation series, Lovell (1986, equation 4) argues that the Mills (1975) *implicit expectations* model could be written in terms of an OLS model of the form

(1) $$F_t = \alpha_0 + \alpha_1 P_t + u_{1,t}$$

where it is assumed that $\alpha_0 = 0$, $\alpha_1 = 1$, $E(u_{1,t}) = 0$ and the predicted error $(u_{1,t})$ is uncorrelated with the actual realization (P_t) $(E(u_{1,t}, P_t) = 0)$. This paper postulates a strong form of the Mills implicit expectations hypothesis (*strong implicit expectations*), which cannot be tested in practice, but which would require the error process of equation (1) to be uncorrelated with the entire information set available to the researcher at time t. A proposed weak form of the implicit expectations model (*weak implicit expectations*), which can be tested in practice, would require that the error process of equation (1) be uncorrelated with past forecasts, $(E(u_{1,t}, F_{t-j}) = 0$ for $j = 1, q)$, and past realizations $(E(u_{1,t}, P_{t-j}) = 0$ for $j = 1, q)$. If one or more of these added assumptions were violated, it would imply that P_t *cannot* be used as a proxie for F_t in empirical work, as was suggested by Mills (1957), since not all information in the forecasted series is in the actual price series in period t. To put this point another way, if given an expected price series F_t that contains *more* information than is contained in our proxie for the actual price series P_t, it would not be permissible to substitute the actual price series for the expected price series as was suggested by Mills. If P_t were found to cause F_t at some frequency, this would cast doubt on the Mills substitution procedure for this problem. By looking at the frequency that violates the Mills procedure, we indirectly obtain information at the frequencies where the Mills substitution is appropriate and thus obtain information on the quality of the expected price series.

Lovell (1986) argues that the Muth (1961) *rational expectations* model requires that the forecast error of equation (2), $(u_{2,t})$, be distributed independently of the anticipated value (F_t). This approach reverses the orthogonality condition of Mills. According to Lovell (1986, equation 5A), Muth's model in OLS terms is

$$(2) \qquad P_t = \beta_0 + \beta_1 F_t + u_{2,t},$$

where $\beta_0 = 0$, $\beta_1 = 1$, $E(u_{2,t}) = 0$ and the error is uncorrelated with the forecasted or anticipated value (F_t) $(E(u_{2,t}, F_t) = 0)$. The strong form of the rational expectations hypothesis (*strong rational expectations*) is difficult to test in practice, since it requires that the errors of the equation (2) $(u_{2,t})$ be "uncorrelated with the entire set of information that is available to the forecaster at the time the prediction is made" (Lovell (1986) page 113). The weak form of the rational expectations hypothesis (*weak rational expectations*), which can be tested in practice, requires that the error process $(u_{2,t})$ be uncorrelated with "historical information on prior realizations of the variable being forecast," (Lovell (1986) page 113)[4] $(E(u_{2,t}, P_{t-j}) = 0$ for $j = 1, q)$ and historical values of the forecasted variable (F_t) $(E(u_{2,t}, F_{t-j}) = 0$ for $j = 1, q)$. If the expected price series were found to cause the actual price series, at some frequency, this would violate the Muth rational expectations hypothesis which assumed that β_1 was equal to one and that errors of equation (2) were uncorrelated with the entire information set available to the researcher.

Lovell makes the important point that under the Muth hypothesis concerning expectations, the variance of P_t is $>$ than the variance of F_t, while under the Mills hypothesis, the reverse holds. Since *Weak rational expectations* requires that the prediction error be uncorrelated with historical information of prior realizations of the variable being forecast, if b_2 in equation (3) is significant, the weak rational expectations hypothesis will be violated.

$$(3) \qquad P_t = b_0 + b_1 F_t + b_2 P_{t-1}.$$

It is important to note that equation (3) can only be used to reject the weak rational expectations hypothesis, since just because b_2 is not significant does not in and of itself rule out that for some other lag, P_{t-k} would be significantly related to the forecast error of equation (3). To do a more complete test of either of the weak implicit or the weak rational expectations hypothesis requires a general model of the form of equation (4) and (5)

$$(4) \qquad F_t = Ei^*(B)F_t + Ci^*(B)P_t + u_t$$

$$(5) \qquad P_t = Gi^*(B)P_t + Hi^*(B)F_t + v_t,$$

where $Ei^*(B)$ and $Gi^*(B)$ are polynomials in the lag operator B for lags $1, \ldots, q$ and $Ci^*(B)$ and $Hi^*(B)$ are polynomials in the lag operator B for lags $0, \ldots, q$.

Equations (4) and (5) are generalizations of equations (1) and (2). Significant terms in Ci^* for lags $1 \ldots q$ would reject the weak implicit expectations hypothesis and call into question the suggestion of Mills (1957) that P_t can be used as a proxie for F_t in empirical work. In equation (5), a finding of significant terms in $Hi^*(B)$ would reject the weak rational expectations hypothesis, since it would imply that the error term of a simpler model such as (2) is *not* orthogonal to prior information (in this case prior expectations values). Later in the paper the above arguments are further refined to look at the relationship between P_t and F_t by frequency. The objective will be to test at what frequency the data are consistent with or reject the weak form of the implicit and rational expectations models. Frankel (1982) asserted he was developing a long run measure of inflation. By decomposing equation (4) and (5) into the frequency domain, we can test whether, if it exists, the dynamic relationship between the expected and actual price series is long-run (low-frequency) or short-run (high-frequency). The above section has outlined the theory why equations (4) and (5) provide a means by which to study Frankel's expected price series from the prospective of the implicit (Mills (1957)) or rational (Muth (1961)) expectations hypothesis. In the next section the Geweke (1982a, 1982b, 1984) procedure is outlined and discussed.

The Geweke procedure involves first estimating a vector autoregressive model (VAR) for the series under study and then decomposing the VAR model into the frequency domain to determine the dynamic structure from the frequency perspective. There is much confusion between the concept of the speed of adjustment and the concepts of the long and short run. Although theory suggests that the Frankel expected price series (F_t) is an unbiased predictor (up to some constant) of the actual inflation rate $(E(F_t) = P_t)$, there has been no systematic study of how the two series are dynamically related. This paper proposes a test of Frankel's expected price series F_t, which uses a VAR model to determine at what frequency, if any, there is a causal relationship between F_t and P_t. By decomposing the VAR model in to the frequency domain, we obtain information concerning whether Frankel's expected price series (F_t) is really a long-run (low-frequency) measure of the actual price series (P_t).

It will be argued in this note that a time measure of the long-run (long-time delay) and short-run (short-time delay) is not appropriate for the problem at hand. Of more interest is the question "at what frequency (if any) are the series F_t and P_t related?". Since the market may be better at figuring out a long-run (low-frequency) component of a cycle than a short-run (high-frequency) component of a cycle, it is important to have a means by which to distinguish between the two components. The question to answer concerns what proportion of the variance at frequency μ of series P_t is captured by series F_t? In addition, if a relationship between F_t and P_t exists, can anything be said about the direction of causality? The information necessary to answer some of these questions is already contained in the VAR coefficients, although it is not apparent from direct inspection. The advantage of the Geweke procedure is that it allows recovery of the frequency information contained in the VAR model and, in addition, via a bootstrap procedure, estimates bounds on the estimated frequency measures.

After first outlining briefly how the Frankel expected price series F_t is calculated, the Geweke procedure is discussed. Next follows a short discussion of various alternatives for the P_t series. Finally, the empirical results are presented and discussed.

3. ECONOMETRIC METHODOLOGY

Frankel's expected price series F_t is developed by assuming that for a given term to maturity, the interest rate reflects a weighted average of an "instantaneously short-term interest rate that is sensitive to the current tightness of monetary policy, and an infinitely long-term interest rate that reflects only the expected inflation rate."[5] The trick is to obtain an estimate of these weights, which were shown by

Frankel (1982) to be a function of speed of the adjustment of a macroeconomic system in discrete and continuous time (in Frankel's notation β and δ, respectively). As Frankel notes, the problem is that the theory underlying the construction of F_t does not work perfectly and, given different points on the term structure, various possible combinations of weights are possible.[6] Rather than attempting to develop new estimates of F_t, this note takes as given Frankel's estimated expected price series and measures, using the Geweke procedure, the frequency information contained in this series in comparison with a representative actual price series. Before a discussion of the appropriate form of the actual price series to use, a few comments on the Geweke (1982a, 1982b, 1984) procedure are in order.

A linear time series process can be represented as a VARMA model

$$(6) \qquad A(B)Z_t = D(B)e_t,$$

where Z_t is a column vector of k random variables and $A(B)$ and $D(B)$ are each k by k matrices whose elements are finite polynomials in the lag operator B. It is assumed that the determinantal polynomials $\det(A(B))$ and $\det(D(B))$ are outside the unit circle so that the VARMA model in equation (6) can be written as a VAR model

$$(7) \qquad \Phi(B)Z_t = e_t,$$

where $\Phi(B) = (D(B))^{-1}A(B)$. If k is assumed to equal 2, where the first element of Z_t is F_t (Frankel's expected price series) and the second element is P_t (a suitably chosen actual price series), the only assumption needed to estimate the VAR model is the maximum order (q) of the VAR polynomial $\Phi(B)$. In the Geweke procedure, it is not clear how best to determine the maximum order q of the polynomial in $\Phi(B)$. In this paper a two-part procedure is employed. First, the SCA[7] system is used to estimate a VAR model of the form of equation (7), using OLS. Inspection of the autocorrelations and cross correlations of the residuals will indicate if q is sufficiently large to summarize the information contained in the Z_t vector. Once the appropriate q is found, the Geweke procedure can be applied safely,[8] since all the systematic information in Z_t will be captured in $\Phi(B)$.

Equation (7) can be written as

$$(8) \qquad Z_t = \Phi^*(B)Z_t + e_t, \qquad Var(e_t) = \Sigma^*,$$

where $\Phi^*(B)$ has no zero-order elements $(\Phi^*(B) = -\Phi(B) - I)$. Following Geweke, equation (8) can be broken up for the two series in Z_t as

$$(9) \qquad F_t = E1^*(B)F_t + u1_t, \qquad Var(u1_t) = \Sigma1,$$

$$(10) \qquad P_t = G1^*(B)P_t + v1_t, \qquad Var(v1_t) = \Gamma1,$$

or as transfer functions of the form

$$(11) \qquad F_t = Ei^*(B)F_t + Ci^*(B)P_t + ui_t, \qquad Var(ui_t) = \Sigma i,$$

$$(12) \qquad P_t = Gi^*(B)P_t + Hi^*(B)F_t + vi_t, \qquad Var(vi_t) = \Gamma i,$$

where if $i = 2$, the transfer function models contain only first-order or greater lags, and if $i = 3$, the transfer models contain zero-order lags in $Ci^*(B)$ and $Hi^*(B)$. If $i = 3$, equations (11) and (12) repeat equations (4) and (5) above. Since $\det(\Sigma1) \geq \det(\Sigma2) \geq \det(\Sigma3)$ and $\det(\Gamma1) \geq \det(\Gamma2) \geq \det(\Gamma3)$, feedback from Y to X[9] $F(Y$ to $X)$ can be defined as $\ln(\det(\Sigma1)/\det(\Sigma2))$ and feedback from X to Y $F(X$ to $Y)$ can be defined as $\ln(\det(\Gamma1).\det(\Gamma2))$.[10] Instantaneous feedback $F_{X,Y} = F_{Y,X} = \ln(\det(\Sigma2)/\det(\Sigma3))$, while linear dependence $F_{X,Y}$ becomes

$$(13) \qquad F(Y \text{ to } X) + F(X \text{ to } Y) + F_{X,Y} = \ln(\det(\Sigma1) * \det(\Gamma1)/\det(\Sigma^*)).$$

Geweke decomposes the above measures into the frequency domain[11] such that $(f(Y$ to $X)(\mu))$

measures the feedback from Y to X at frequency μ and $(f(X$ to $Y)(\mu))$ measures the feedback from X to Y at frequency μ.

Although there is a one-to-one mapping from the VAR model estimated in equation (7) and the frequency feedback measures, it is important to access the "significance" of an estimated measure of feedback at frequency μ. Geweke's proposed solution is to perform a bootstrap procedure, where τ sets of Monti Carlo data are generated with the asymptotic distribution of $\Phi(B)$ being set equal to the actual estimated value from the data sample. If the frequency decomposition is performed τ times, approximate confidence intervals can be calculated for the estimates, which can be adjusted for small sample bias. The end result is that from this procedure we can determine the frequency relationship between X and Y that is implicit in $\Phi(B)$ in equation (11) and (12). For the purposes of this study, τ was set at 100. In the next section, we will discuss the alternatives for the price series P_t and proceed to present the findings.

4. FINDINGS

The Frankel (1982) expected price series, F_t, runs from 1959/7 to 1979/4 (237 observations). The selection of an actual price series to relate to this series is not unique. It was decided that a reasonable choice to use was the consumer price index-urban, which was obtained from the August 1982 version of the NBER/Citybase data tape for the period 1954/7 to 1979/4. Three transformations used in the empirical work. If we assume that P_t is the raw CPI-U series, then DIFPCPZU $= (1 - B) * 12 * ((P_t - P_{t-1})/P_{t-1})$ is a first difference of the annualized percent change in P_t. This transformation is comparable to a first difference of the Frankel expected long-run inflation rate, DIFFPRCE $= (1 - B) * F_t$. (Both series were differenced once to achieve stationarity.) An argument can be made that since the Frankel series is an expected *long-run* inflation rate, the appropriate actual series to use would involve calculation of the percent change over a longer period. To deal with this concern, two additional transformations of P_t were tried. An intermediate measure, DIFPCPZU1 $= (1 - B)((P_t - P_{t-12})/P_{t-12})$ annualizes on the basis of a year, while a more long-term measure, DIFPCPZU5 $= (1 - B) * (.2) * ((P_t - P_{t-60})/P_{t-60})$, annualizes on the basis of a five-year horizon.[12] In all VAR estimations, the same data period was used, 1959/9 to 1979/4, to facilitate comparisons among alternative transformations of the price series. As was discussed earlier, the maximum order of the polynomial $(q = 12)$ in $\Phi(B)$ was selected from inspection of autocorrelations of equation (7) estimated by OLS.[13]

The results from estimating three models are given in Tables 1–3 and will be discussed in turn. In Table 1, the results for the annualized percent change in the CPI-U (DIFPCPZU) indicate strong (32.9%) high-frequency (period = 3) feedback from the price series to the expected price series. This indicates that at this frequency (high) the predicted error of equation (1) is not uncorrelated with the entire set of information that was available and thus that at this frequency the weak form of the implicit expectations hypothesis does not hold. In terms of our prior notation, we have significant terms in Ci^* in equation (4). The only other feedback found of any note is at somewhat lower frequencies, where for periods equal to 8, 7, and 6, the percent explained was 8.0%, 11.8%, and 13.0%, respectively, for the estimates corrected for small-sample bias.

There is less evidence of lags of the expected price series mapping to the actual price series; the only evidence of any relationship was found at lower frequencies. For example, at periods equal to 60, 40, 32, 16, and 12, the percentages explained are only 6.3%, 8.2%, 9.4%, 11.5%, and 8.0%, respectively. Overall, there was more of a relationship from P_t to F_t $(F(Y$ to $X) = 5.0\%)$ than from F_t to P_t $(F(X$ to $Y) = 3.1\%)$. We can summarize Table 1 by noting that the feedback from the actual price series to the expected series was at a high frequency and much stronger than the much lower low frequency mapping from the expected price series to the actual price series. These results are more consistent with the weak form of the rational expectations model than the weak form of the implicit expectations model. The findings support Frankel's contention that his expected price measure was a long-run (low-frequency in our terms) measure.

TABLE 1
Estimated Measures of Linear Feedback Annualized Data

	X vector DIFFPRCE				Y vector DIFFCPZU			
		Estimate			Adjusted Estimate		25%	75%
F(Y to X)		.075 (7.3%)			.051 (5.0%)		.040	.059
F(X to Y)		.059 (5.7%)			.031 (3.1%)		.024	.037
F(X.Y)		.009 (0.9%)			.005 (0.5%)		.001	.007
		f(Y to X)				f(X to Y)		
Period	Estimate	Adj. Est.	25%	75%	Estimate	Adj. Est.	25%	75%
120	.018 (1.8%)	.008 (0.8%)	.001	.010	.076 (7.3%)	.048 (4.6%)	.011	.069
60	.018 (1.8%)	.009 (0.9%)	.002	.010	.093 (8.8%)	.065 (6.3%)	.024	.090
40	.019 (1.9%)	.009 (0.9%)	.002	.011	.112 (10.6%)	.086 (8.2%)	.035	.116
32	.020 (2.0%)	.010 (1.0%)	.002	.012	.126 (11.8%)	.099 (9.4%)	.044	.134
16	.026 (2.5%)	.011 (1.1%)	.004	.014	.154 (14.3%)	.122 (11.5%)	.065	.168
12	.034 (3.3%)	.016 (1.6%)	.006	.024	.118 (11.1%)	.084 (8.0%)	.036	.113
10	.052 (5.1%)	.032 (3.2%)	.015	.047	.067 (6.5%)	.037 (3.7%)	.013	.049
8	.104 (9.9%)	.083 (8.0%)	.042	.108	.055 (5.4%)	.029 (2.9%)	.013	.039
7	.151 (14.0%)	.125 (11.8%)	.073	.162	.087 (8.3%)	.057 (5.5%)	.023	.076
6	.183 (16.7%)	.140 (13.0%)	.075	.186	.097 (9.2%)	.067 (6.5%)	.030	.094
5	.052 (5.1%)	.022 (2.2%)	.009	.030	.021 (2.1%)	.007 (0.7%)	.002	.010
4	.006 (0.6%)	.001 (0.1%)	.000	.001	.044 (4.3%)	.019 (1.8%)	.005	.027
3	.393 (32.5%)	.399 (32.9%)	.216	.531	.013 (1.3%)	.002 (0.2%)	.001	.003
2	.001 (0.1%)	.000 (0.0%)	.000	.000	.119 (11.2%)	.069 (6.7%)	.012	.117
Inf	.018 (1.8%)	.008 (0.8%)	.001	.011	.069 (6.7%)	.040 (4.0%)	.006	.060
Mean %	6.61%	5.22%			7.93%	5.33%		
SD of %	8.71%	8.74%			3.66%	3.22%		

See text for further discussion of variable labels and method. B = lag operator. DIFFPRCE = $(1 - B)F_t$, where F_t = Frankel expected price series.

Mean = .16864407E-03   SD = .19896626D-02

DIFPCPZU = $(1 - B) * 12 * ((P_t - P_{t-1})/P_{t-1})$, where P_t = CPI-U all items NSA.

Mean = .64172852E-03   SD = .32871276D-01

Table 2, where the yearly difference of the CPI-U (DIFPCPZU1) is used, presents somewhat of a different frequency pattern, as might be expected, given the different actual price series used. The overall measures F(Y to X) and F(X to Y) are larger than in Table 1 (6.2% vs 5.0% and 4.4% vs 3.1%, respectively). In contrast to Table 1, where the average percentages for f(Y to X) and f(X to Y) were 5.22% and 5.33%, respectively (for adjusted estimates), in Table 2 the corresponding percentages are 13.25% and 6.03%, respectively.[14] In comparison to Table 1, we see large low-frequency feedback, which would indicate rejection of the weak form of the implicit expectations hypothesis at low frequencies. For example, at periods equal to 60, 40, and 32, the percent explained is 29.1%, 67.3%, and 52.2%, respectively. Going the other way (f(X to Y)), there is little evidence of a relationship, except for relatively low values at periods equal to 16, 7, and 6, where the percentages are 12.3%, 12.5%, and 19.9%, respectively. It is worth noting that the effect found at 16 for f(X to Y) is relatively similar across the two tables. It is apparent that the low-frequency findings appear to be sensitive to the construction of the actual price series. The finding that the relationship from P_t to F_t dominates the relationship from F_t to P_t is similar to that in Table 1. Table 2, like Table 1, is consistent with the Muth weak rational

TABLE 2
Estimated Measures of Linear Feedback One-Year Data

	X vector DIFFPRCE				Y vector DIFFCPZU1			
		Estimate			Adjusted Estimate		25%	75%
F(Y to X)		.089 (8.5%)			.064 (6.2%)		.050	.077
F(X to Y)		.074 (7.2%)			.047 (4.5%)		.036	.054
F(X.Y)		.018 (1.8%)			.014 (1.4%)		.004	.022
		f(Y to X)				f(X to Y)		
Period	Estimate	Adj. Est.	25%	75%	Estimate	Adj. Est.	25%	75%
120	.105 (10.0%)	.108 (10.2%)	.042	.146	.015 (1.5%)	.004 (0.4%)	.001	.005
60	.300 (25.9%)	.343 (29.1%)	.191	.487	.037 (3.6%)	.019 (1.9%)	.008	.024
40	.858 (57.6%)	1.117 (67.3%)	.598	1.60	.065 (6.3%)	.047 (4.6%)	.025	.058
32	.686 (49.7%)	.739 (52.2%)	.350	.992	.087 (8.3%)	.071 (6.8%)	.041	.090
16	.091 (8.7%)	.075 (7.2%)	.038	.099	.148 (13.8%)	.132 (12.3%)	.068	.170
12	.066 (6.4%)	.050 (4.9%)	.026	.068	.099 (9.5%)	.083 (8.0%)	.034	.114
10	.077 (7.4%)	.055 (5.3%)	.032	.075	.032 (3.1%)	.015 (1.4%)	.006	.020
8	.153 (14.2%)	.101 (9.6%)	.041	.136	.063 (6.1%)	.038 (3.8%)	.013	.055
7	.022 (2.2%)	.008 (0.7%)	.003	.011	.163 (15.0%)	.133 (12.5%)	.061	.194
6	.006 (.6%)	.001 (0.1%)	.000	.002	.262 (23.0%)	.222 (19.9%)	.122	.294
5	.009 (.9%)	.000 (0.0%)	.000	.001	.136 (12.7%)	.104 (9.9%)	.053	.147
4	.013 (1.3%)	.005 (0.5%)	.002	.006	.008 (0.8%)	.001 (0.1%)	.000	.001
3	.049 (4.8%)	.036 (3.5%)	.014	.045	.096 (9.1%)	.053 (5.2%)	.021	.067
2	.031 (3.0%)	.017 (1.6%)	.002	.024	.064 (6.2%)	.036 (3.6%)	.004	.045
Inf	.068 (6.6%)	.068 (6.6%)	.009	.104	.006 (0.6%)	.001 (0.1%)	.000	.001
Mean %	13.29%	13.25%			7.97%	6.03%		
SD of %	17.66%	20.40%			6.18%	5.65%		

See text for further discussion of variable labels and method. B = lag operator. DIFFPRCE = $(1 - B)F_t$, where F_t = Frankel expected price series.

Mean = .16864407E-03   SD = .19896626E-02

DIFPCPZU1 = $(1 - B)((P_t - P_{t-12})/P_{t-12})$, where P_t = CPI-U all items NSA.

Mean = .40832558E-03   SD = .31466113E-02

expectations hypothesis. Unlike Table 1 where the weak implicit expectations hypothesis was rejected at high frequencies, here the rejection is at low frequencies.

Table 3 utilizes the longest run measure for P_t (DIFPCPZU5). The findings here are more similar to those found in Table 1 than in Table 2. The strongest feedback was found at a very low frequency (at period = 120, the percent is 10.7%), at several intermediate frequencies (at periods = 12, 10, 8, 7, and 6, the percentages were 7.0%, 11.1%, 17.2%, and 14.0%, respectively) and at one very high frequency (at Inf the percentage is 11.7%). Going the other way (i.e., f(X to Y)), there is little evidence of a relationship except for small effects at periods 16, 12, and 10, where the percentages are 8.0%, 8.4%, and 6.3%, respectively. The summary measures F(X to Y) and F(Y to X) are less than in either Table 1 or Table 2, but follow the same pattern (i.e., F(X to Y) < F(Y to X)). The instantaneous effect (F(X.Y)) between the two series increased from Table 1 to Table 3 (.5%, 1.4%, and 2.0%, respectively) as the length of the lag in generating the appropriate P_t variable increased from 1 to 60. Like the prior two tables, the results in Table 3 are more consistent with the weak form of the Muth rational expectations hypothesis and relatively less consistent with the weak form of the Mills implicit expectations hypothesis.

TABLE 3
Estimated Measures of Linear Feedback Five-Year Data

X vector DIFFPRCE		Y vector DIFFPCPZU5			
	Estimate	Adjusted Estimate		25%	75%
F(Y to X)	.062 (6.0%)	.035 (3.4%)		.027	.040
F(X to Y)	.034 (3.3%)	.014 (1.4%)		.010	.017
F(X.Y)	.023 (2.3%)	.021 (2.0%)		.009	.027

	f(Y to X)				f(X to Y)			
Period	Estimate	Adj. Est.	25%	75%	Estimate	Adj. Est.	25%	75%
120	.165 (15.2%)	.114 (10.7%)	.022	.180	.006 (0.6%)	.001 (0.1%)	.000	.001
60	.128 (12.0%)	.085 (8.2%)	.023	.125	.017 (1.7%)	.005 (0.5%)	.002	.006
40	.101 (9.6%)	.064 (6.2%)	.021	.090	.035 (3.4%)	.016 (1.5%)	.006	.019
32	.088 (8.4%)	.055 (5.3%)	.019	.075	.051 (5.0%)	.027 (2.7%)	.010	.034
16	.071 (6.8%)	.051 (5.0)%	.024	.067	.125 (11.7%)	.083 (8.0%)	.036	.107
12	.092 (8.7%)	.073 (7.0%)	.035	.100	.126 (11.8%)	.088 (8.4%)	.037	.123
10	.135 (12.7%)	.118 (11.1%)	.061	.155	.099 (9.5%)	.065 (6.3%)	.030	.087
8	.215 (19.3%)	.189 (17.2%)	.092	.268	.043 (4.2%)	.020 (2.0%)	.005	.027
7	.188 (17.1%)	.151 (14.0%)	.076	.222	.013 (1.3%)	.003 (0.3%)	.001	.003
6	.128 (12.0%)	.090 (8.6%)	.027	.137	.002 (0.2%)	.000 (0.0%)	.000	.000
5	.060 (5.8%)	.028 (2.8%)	.009	.040	.006 (0.6%)	.001 (0.1%)	.000	.001
4	.014 (1.4%)	.004 (0.4%)	.001	.006	.008 (0.8%)	.001 (0.1%)	.000	.001
3	.045 (4.4%)	.018 (1.8%)	.006	.027	.025 (2.5%)	.008 (0.8%)	.002	.011
2	.001 (0.1%)	.000 (0.0%)	.000	.000	.050 (4.8%)	.031 (3.0%)	.005	.045
Inf	.187 (17.0%)	.124 (11.7%)	.014	.211	.002 (0.2%)	.000 (0.0%)	.000	.000
Mean %	10.03%	7.33%			3.89%	2.25%		
SD of %	5.76%	5.02%			4.05%	2.95%		

See text for further discussion of variable labels and method. B = lag operator. DIFFPRCE = $(1 - B)F_t$, where F_t = Frankel expected price series.

Mean = .16864407E-03   SD = .19896626E-02

DIFPCPZU5 = $(1 - B) * .2 * ((P_t - P_{t-60})/P_{t-60})$, where P_t = CPI-U all items NSA.

Mean = .32661320E-03   SD = .75361922E-03

It must be stressed that the intention of the paper is to investigate the dynamic relationship between the proposed Frankel expected price series and three actual price series, *not* to test the Muth or Mills expectations hypothesis. To effectively perform the latter tests would require a different econometric setup and would require jointly estimating and testing the Frankel series with appropriate cross equations restrictions. The more modest goal of this paper has been to take the published Frankel series as a given, relate this series to three proposed price series, and give some possible economic content to the frequency at which relationships are found.

5. CONCLUSIONS

In an innovative article, Frankel (1982), utilizing relationships implicit in the term structure of interest rates, developed a long-run measure of expected inflation. The Frankel expected price series is causally related, by frequency, to three of many possible transformations of the consumer price index (urban). In most cases it was found that, for the frequencies where there was a relationship, the actual price series caused the expected price series, which is not consistent with the assumptions of the Mills weak implicit expectations hypothesis. This finding casts doubt on Mills proposal to use the actual price series as a proxie for the expected price series F_t for that frequency. This finding must be qualified, since from theory it is not clear which actual series the expected price series is supposed to proxie for. The instantaneous relationship between the expected price series and the actual price series appears to be quite low.

NOTES

1. This paper uses the Granger (1969) definition of causality which states that X_t causes Y_t if and only if for a given an information set, which includes at least X_t and Y_t, Y_t can be predicted better by using past X_t than by not using it.
2. Frankel (1982) footnote 14 mentioned that he had used his expected measure of inflation to test the hypothesis of rational expectations. His research, only briefly mentioned in his 1982 paper, rejected the null hypothesis.
3. In a later section, there will be a discussion of the appropriate series to use for P_t and F_t. In the empirical work, three alternative specifications are reported. The discussion of the relationship between implicit and rational expectations follows the important survey article by Lovell (1986).
4. Lovell (1986) discussed the strong and weak form of the rational expectations hypothesis. This paper proposes the strong and weak form of the implicit expectations hypothesis.
5. Frankel (1982) page 136.
6. In footnote 11, Frankel (1982) outlines a possible cross-section time series procedure to determine the weights and correct for possible liquidity effects, which are assumed away here.
7. Since no constraints are being placed on $\Phi(B)$, OLS can be used. SCA, which was developed by Liu and Hudak (1983), was used for the VAR estimates in the identification stage.
8. Because of excessive computational cost during the bootstrap procedure, which will be discussed later, the Geweke procedure uses the Whittle (1963) approach to calculate the estimates of $\Phi(B)$ rather that OLS. Stokes (1985) discusses some of the costs of this approach. Due to software limitations, in this paper the Geweke procedure has been followed. (Calculations have been made using a modified version of Geweke's code. The author is grateful to Professor Geweke for making this code available. The author has made this code run under SAS® Software as part of the B34S® Data Analysis program.)
9. For the purposes of the discussion, we define the X series as the first series in the Z_t vector, F_t in this case, and the Y series as the second series in Z_t, P_t in this case. The reason for the switch in the notation is to emphasize the fact that, while in the present case there is only one X series and one Y series, in the general case there can be more than one X and Y series. If there are more than one X and Y series, equations (9)–(12) are VAR models. In the case where there is one X series and one Y series, equations (9)–(12) are both transfer functions and VAR models.
10. As Geweke has noted, the motivation for these definitions lies in the fact that the natural log of 1 is zero and the fact these measures can be interpreted in terms of the one-step-ahead population errors. For added detail see Stokes (1991) which documents the B34S Data Analysis Program used to make the calculations.
11. The details of the decomposition will be skipped in this brief note. For further details see Geweke (1982a). Although both the unadjusted and adjusted measures of feedback are reported, only the adjusted measures are discussed in the text because these measures are corrected for small sample bias.
12. A transformation involving a 6-month lag, or some multiple of a 6-month lag, was not attempted because of possible additional complications involving the seasonal in the price series.
13. An OLS estimate of $\Phi(B)$ in equation (7), where Z_t = DIFFPRCE, and P_t = DIFPCPZU, and assuming q = 12 with 24 lags of the residuals autocorrelated and cross correlated, indicated only 4 marginally significant spikes out of 96 (24 * 4) possible cross correlations and autocorrelations calculated. These spikes were found at lag 14 ($p_{1,2}$ = .17), lag 15 ($p_{1,2}$ = .16), lag 18 ($p_{1,1}$ = −.14) and lag 19 ($p_{2,2}$ = −.14). In all subsequent work involving transformations of P_t, q was assumed equal to 12. This method of determining the value to assume for q is in keeping with current practice in the VAR time series literature. It is to be noted that the OLS estimates of $\Phi(B)$ will not agree with the Whittle (1963) procedure estimates of $\Phi(B)$, although they are asymptotically equivalent. For further discussion of the difference between these alternative estimates, see Stokes (1985) footnote 10. In that footnote it was noted that there were differences in the Whittle procedure obtained in the IBM version of Geweke's program and on the DEC version. Since Stokes (1985), Geweke, in a private letter, has informed me that there was an error in the DEC results. This leaves only the question of the magnitude of the difference between the Whittle (1963) estimates of $\Phi(B)$ and the OLS estimates. This is a topic for further research.
14. The average percentage for the periods is admittedly only a summary measure. Its value is dependent on the periods used for the estimation.

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