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Real Money Balances in the Production Function: A Comment

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Jensen-Kamath-Bennett (JKB) (1987) and Jensen-Kamath (JK) (1987) argue that the Sinai-Stokes (SS) (1972) empirical results on money balances in production are questionable because a proposed counterexample specified and estimated by them also is "confirmed" by the data.

While it is appropriate to propose a counterexample to question the validity of a maintained hypothesis, a difficulty of the JKB approach is that the two counterexample hypotheses estimated, and reported again in JK, are just forms of the same counterexample, which itself is a special case of the original SS result. For a counterexample to work in the manner intended by JKB, it must be logically independent of the example, *not* a restricted form of the more general functional specification.

SS (1972) tested a Cobb-Douglas (CD) production function with three inputs: labor, capital, and money balances. In addition, a time trend was entered in some of the equations estimated. In SS (1972), the elasticities of output with respect to the inputs were *not* constrained to add to unity. JKB constrained the sum of the output elasticities with respect to the inputs at unity.

Given Q as output; t, a time trend; and K, L, m capital, labor and real balances, respectively, SS estimate

$$\ln Q = \ln A + \Phi t + \alpha \ln L + \beta \ln K + \tau \ln m + u \qquad (1)$$

using annual time series data over 1929-1967. The basic SS results for equations containing a time trend were provided in SS (1972) (Equations 8, 9) and again reported in SS (1989) (Equations 9, 14, Table 7). JKB estimate

$$\ln (Q/m) = \ln A + \Phi t + \alpha \ln (L/m) + \beta \ln (K/m) + u', \qquad (2)$$

which is nothing more than a constrained version of (1).[1] Equation (2) is not a counterexample of (1); it is a special case. For the counterexample methodology used by JKB to be appropriate, it must be *logically* distinct from the example. The other counterexample proposed by JKB is econometrically the same as (2).

There are two ways to estimate a constrained three-input production function and force the coefficients to add to unity. If two of the three inputs in a CD production function are divided by the other input and the result estimated by ordinary least squares (OLS), constant returns to scale are assumed. This was done in (2). The other way is to estimate (1) with a restriction on the coefficients to sum to one. Both approaches will give the results shown in Table 2a of JKB, who indicate support for their counter- example because equations containing ln M1, ln M2 and ln M3 provide a "GOOD-FIT" (JKB, p.265), while the "original" SS model shows a "GOOD-FIT" for models containing ln M1 and ln M2, and a "BAD-FIT" for the equation containing ln M3.

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What is a "good-fit" and "bad-fit" is not indicated. The results in Table 2a actually show that a better fit is obtained for all the SS unconstrained equations compared with the unrestricted and restricted JKB equations. The only criticism of the SS results reported by JKB might be the low t-statistic on the coefficient of M3 (1.346). The negative coefficients for M1, M2 and M3 in the JKB constrained case are forced by the form JKB used.

Our interpretation of the JKB results is that the effect of imposing the restriction on (1) is to lower the $\bar{R}^2$ in all equations, providing evidence that the special constant returns to scale case of the SS increasing returns CD specification is an incorrect specification on the data used, rather than a counter- example supported by the data.

For example, using the JKB data, for the M1 regression, the overall fit falls from 0.99229 to 0.95146, for M2 from 0.99035 to 0.93652, and for M3, from 0.9896 to 0.94688. Far from the sweeping conclusion (JKB, p. 259), "we show that all the evidence produced in support of the Sinai-Stokes hypothesis cannot be sustained on theoretical and empirical considerations," what shows is that the constrained three-input CD production function gives an inferior fit compared with the unconstrained form estimated by SS.[2] This JKB conclusion is an incredible overstatement on the results presented.

JKB fault SS (1972) for not checking on multicollinearity when, in fact, such a check was performed by running all prospective variables on already included variables and checking the R^2. These results were calculated using the B34S™ program, documented in Stokes (1991) and has a number of collinearity tests on the regression calculation, including the Faddeeva (1959) procedure, which provides a computational check on the estimated coefficients.

While there was some multicollinearity in the data, the problem was not particularly serious. The Faddeeva estimates of the computational error in each coefficient for the OLS form of (1) for M1 and M2 are (0.5299E-11, -0.4894E-11), (0.1064E-10, -0.3142E-11), (0.3979E-10, 0.18929E-10), (-0.1767E-10, 0.7824E-11), and (-0.8011E-11, 0.1580E-11) for ln A, Φ, α, β and τ, respectively. Given that X is the T by K data matrix, the Faddeeva procedure calculates the diagonal elements of $(X'X(X'X)^{-1}\text{-}I)$, which should have an expected value of zero. Substantial deviations from zero provide a computational check on the accuracy of the answers. Since the largest diagonal element found was in the area of 0.4E-10, accuracy loss due to multicollinearity does not appear to have been a serious problem. Actually, any multicollinearity in the data set, by inflating the diagonal elements of the variance-covariance matrix, would work in the direction of reducing the estimated significance for the coefficient of real money balances.

Further evidence on whether money balances belong in the production function awaits implementation of more comprehensive measures for the role of the financial system in production and additional experimentation with functional form issues. While we applaud the development of the "counterexample" methodology as a means by which to test and study the effect of the financial system on the potential supply of real output, such counterexamples must be distinct from the original SS unconstrained CD form in order to have validity. All the JKB paper does is to highlight that the constrained form of the CD production function containing real balances is inferior as a specification to the unconstrained form for the 1929-67 data set used in the estimation. It does not controvert or shed doubt on the original carefully drawn and stated suggestions of the original work.

NOTES

1. Equation 1 comes from taking logs of $Q=Ae^{\Phi t}L^{\alpha}K^{\beta}m^{\tau}v$, where u = ln v. The constrained system is $Q=Ae^{\Phi}tL^{\alpha}K^{\beta}m^{1-\alpha-\beta}v$. Taking anti-logs of equation 2 and multiplying by m, we get $Q=m(Ae^{\Phi}tL^{\alpha}K^{\beta}m^{1-\alpha-\beta}v)$, where v is the anti-log of u'. Hence, equation 2 is just one of two possible ways to estimate a constrained form of equation 1.
2. Benzing (1989) has recently commented on JKB (1987), who again replied in JKB (1989). Benzing used more recent data that was provided to us. She reports the second-order GLS unrestricted form of the SS model and finds that the coefficients of ln M1, ln M2 and ln M3 were significant in models without time. With models containing time, she reported significance in the coefficients of money variables, although there was some evidence of multicollinearity.

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A Counter-counter Critique : A Reply to Sinai - Stokes

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In discussing our critique of their 1972 work, Sinai and Stokes (1992) "approve of the use of a valid counterexample to question the validity of a maintained hypothesis" but then proceed to declare our counterexample as inadmissable because of its being a "special case" of their hypothesis and not *logically distinct* from the (original) example." We are pleased that SS agree with us on the need to recast conventional testing procedures in economics in terms of the counterexample procedure but fear that they do not recognize the methodological implications of their admission and its implications for their hypothesis. We reiterate some old issues and examine the invalidity of their counter-critique in this section.

As pointed out in our original (1984) paper (most recently published as Kamath, Jensen, & Bennett (1990)), the conventional approach to empirical testing, as utilized by SS in their 1972 paper, involves attempting to confirm predictions derived from models of the theory under examination. There are two major problems with this method. Firstly, rules of evidence, which are used to determine if the prediction fits the data, are generally structured such that a negative result does not mean that the model or theory under consideration is false, likewise a positive result does not mean that the model or theory is true.[1] And secondly, despite the possibility of errors analogous to the type I and type II errors of standard hypothesis testing inherent in the usage of all rules of evidence, common test procedures do not reflect the need to minimize these errors or reach an optimal trade-off between them.[2] These characteristics of the conventional approach to testing reduce the efficacy and efficiency of empirical testing in economics.[3] As is well known, a universal theory can be logically contradicted or refuted with the aid of deductive logic by only one singular statement while no amount of confirming statements can ever guarantee its truth.[4] Any test procedure for empirical testing needs to exploit this asymmetry in testing theories.