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Some Milestones in Econometric Computing

# Econometric software as a theoretical research tool

Houston H. Stokes
*Department of Economics, University of Illinois at Chicago, 601 South Morgan Street* (*M/C 144*),
*Chicago, IL 60607, USA*
*E-mail: hhstokes@uic.edu*

It is common that the discovery or development of a statistical finding precedes its implementation, but in certain cases this is not true. The development of the Theil BLUS technique is a specific example of a two-way relationship between econometric theory and software design. In particular, the fact that the sum of squares of the N-K BLUS residuals is equal to the sum of squares of the N OLS residuals was first observed experimentally and subsequently proved in theory.

## 1. Introduction

In the Forward to his classic econometrics textbook, Theil [13, p. 5] suggests, "Nobody believes that he can become a chemist by attending lectures and reading textbooks and journal articles. He should devote time and energy to the real work in the laboratory. In the same way, nobody should believe that he will be able to handle statistical data in econometrics without touching actual data." In the spirit of this thought, the present note outlines how the analysis of data in one case gave rise to new theory, a phenomenon that is common in hard sciences such as physics and chemistry but less so in econometrics.

Prior to writing his textbook, in an important paper Theil [11] had already laid out the theory for the BLUS (best, linear, unbiased, scalar covariance matrix) residuals. The essential objective behind the development of the BLUS concept was to transform the sample OLS residuals in such as way as to provide these new residuals with a scalar covariance matrix–so that exact tests could be performed for a variety of problems such as heteroskedasticity, serial correlation nonlinearity and normality. Theil first proved that there could be at most N-K BLUS residuals, where N is the sample size and K is the number of right hand side variables. The problem then became how best to determine which K residuals to place in the base from which to calculate the N-K BLUS residuals. For each problem, there would be a number of possible bases that should be considered. As a result of detailed empirical analysis, Theil solved this selection problem by developing a structured way to determine the most appropriate base for various statistical tests.

## 2. A brief review of BLUS residual theory

To begin, it might be helpful to recall certain results. Assume $y$ is a N-element left hand side variable, $X$ is a $N$ by $K$ matrix of exogenous variables, $\beta$ is a $K$ by 1 matrix of coefficients and $u$ is the N element population residual. Then consider the linear model:

$$y = X\beta + u \tag{1}$$

The OLS estimate of the coefficient vector is of course

$$\hat{\beta} = (X'X)^{-1}X'y \tag{2}$$

from which we can define the vector of estimated or sample residuals $\hat{u}$ as

$$\begin{aligned}\hat{u} &= y - X\hat{\beta} \\ &= X\beta + u - X(X'X)^{-1}X'y \\ &= X\beta + u - X(X'X)^{-1}X'(X\beta + u) \\ &= u - X(X'X)^{-1}X'u \\ &= (I - X(X'X)^{-1}X')u\end{aligned} \tag{3}$$

The presently important implication of the result shown as Eq. (3) is that even if the population residual $u$ has a scalar covariance matrix, the sample residual will not. As a consequence, the sample residuals will not have this (assumed) ideal property of the original errors.

However, Theil [11] realized that fortuitously there exists a vector $\hat{\varepsilon}_1$ defined for the subset of T-K observations associated with the data sub-matrix $X_1$, that is also an estimator of the population residuals, but which has a scalar covariance matrix. After some work, Theil [12] simplified the estimation of these BLUS residuals to

$$\hat{\varepsilon}_1 = u_1 - X_1 X_0^{-1} \left[ \sum_{h=1}^{K} \frac{d_h}{(1+d_h)} q_h q_h' \right] u_0 \tag{4}$$

as is described in greater detail in Theil [13], where $X_0$ is the $K$ by $K$ matrix of observations in the base and $d_i$ and $q_i$ are the eigenvalues and eigenvectors of the matrix $X_0(X'X)^{-1}X_0'$. The $N-K$ by $K$ matrix $X_1$ is the observations that are not in the base.

## 3. Properties of the BLUS residuals

In his original presentation, Theil [11] does not mention the fact that the sum of squares of the N OLS residuals is the same as the sum of squares of the N-K BLUS residuals. However, two years later, Koerts [3], who worked with Theil in Rotterdam on the calculations, published this result simply as a calculation checksum. By this time Theil had moved to the University of Chicago. Shortly thereafter Thornber [15, 16] had incorporated BLUS calculations into B34T, the then standard econometric estimation program at Chicago, developed by him there. In 1972, Stokes took over development of B34T, which became B34S and in which these calculations were then incorporated.[1]

Subsequently, in 1977, Stokes was asked by Theil to work on a BLUS example of nonlinear detection, while he was in the process of revising his textbook. This work was intended to be included in a revision of Chapter 5 (Theil [14]). During this collaboration Theil told Stokes orally that the result

$$\sum_{i=1}^{T-K} \hat{\varepsilon}_{1i}^2 = \sum_{i=1}^{T} u_i^2 \tag{5}$$

had first been discovered experimentally, and only later proved. This revised version of Chapter 5 was completed, but the complete book was never published. The reason was apparently Theil's unwillingness to cannibalize his earlier work: Kenneth Clements, who worked with Theil in the 70', informed Stokes that in the 1980's Theil told him he had abandoned the revised edition since he would be "competing with himself."

The BLUS example Theil and Stokes developed has however been published, in Stokes [6,7]. Its development involved use of the logic of problem 5.2 in Theil [13] which had been moved into the text of the revised chapter 5 in Theil [14] as a part of this example. The importance of Eq. (5) as a result is a consequence both of its value as a computational check and the fact that it is related to the Theil [13] and Koerts [3][2] result that the minimum expected sum of squares of the N-K BLUS errors is

$$E[(\hat{\varepsilon}_1 - u_1)'(\hat{\varepsilon}_1 - u_1)] = 2\sigma^2 \sum_{i=1}^{K} (1 - d_i). \tag{6}$$

Inasmuch as Theil [12] proved that the maximum eigenvalue was at most 1.0, it follows immediately that Eq. (6) directly implies that the expected error of the T-K BLUS estimates of unobserved population vector $u_1$, for the non-base observations, will be minimized the larger is

$$\sum_{i=1}^{K} d_i \tag{7}$$

[1] Stokes [8,9] documents the B34S whose history and development is discussed in Stokes [10] in the context of Renfro [7].

[2] See Theil [13] Theorem 5.4. A slightly different formulation is given in Koerts [3].

thus forming the theory basis for using the sum of the square roots of the eigenvalues as the BLUS selection criteria whenever there is a possibility of alternative appropriate bases ($X_0$).

## 4. Summary

The BLUS technique continued to be listed in econometric books such as Chow [1] and Maddala [5] into the 1980's. Theil [14], although never published, contained a response to Koerts and Abrahamse [4] that had questioned the power of the BLUS test. At issue was the use of a non-optimum BLUS base. In Theil's view and the experience of Stokes [8,9], it is a serious error not to utilize Eq. (7) to select the appropriate BLUS base. It remains to be seen if errors in BLUS base selection is the cause of some of the lack of power in the BLUS procedure found by Huang and Bolch [2] and Ramsay [6] and cited in Maddala [5].

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